Diffusion of Carriers

Whenever there is a ***concentration gradient of mobile particles, they will diffuse from the regions of high concentration to the regions of low concentration, due to the random motion***. The diffusion represents an important charge transport process in semiconductors.

Carriers in a semiconductor diffuse in a carrier gradient by *random thermal motion and scattering from the lattice and impurities*.

As the electrons (or holes) move with a thermal velocity v_{th} they undergo random collisions. *In the absence of an electric field they have an equal probability of moving in any direction in between collisions*.

Average distance travelled between collision is the ***mean free path***, ***l***
Average time between collisions is the ***mean free time***, τ_c

$$l = v_{th}\tau_c$$

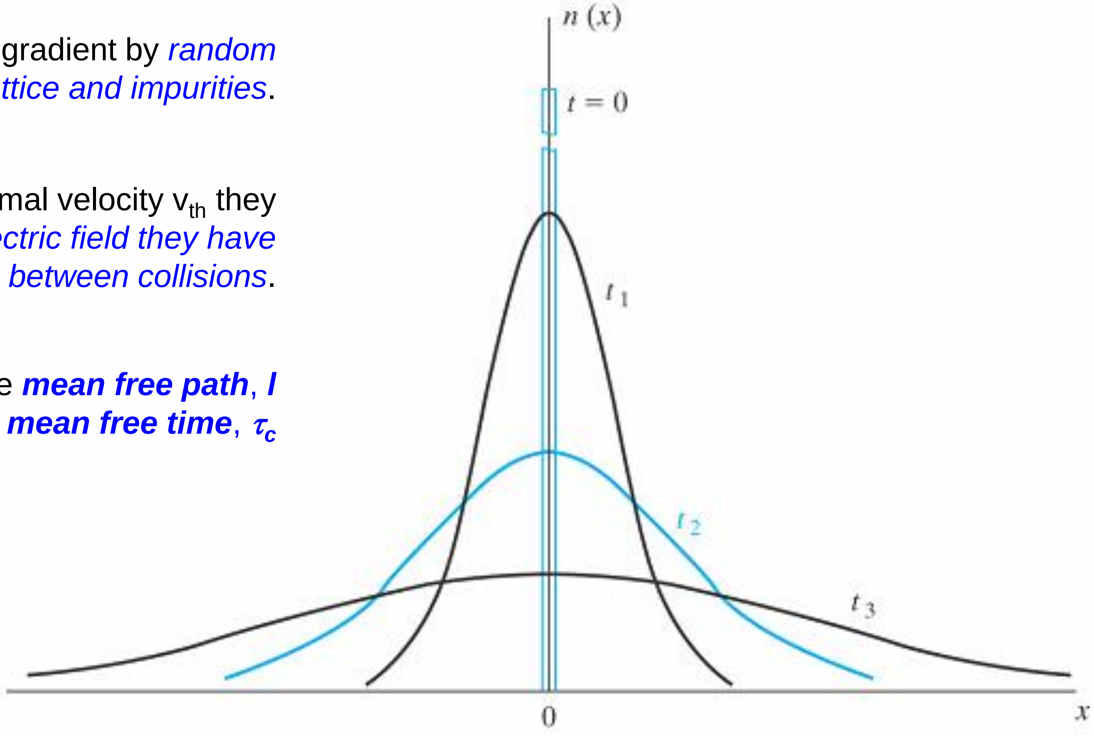
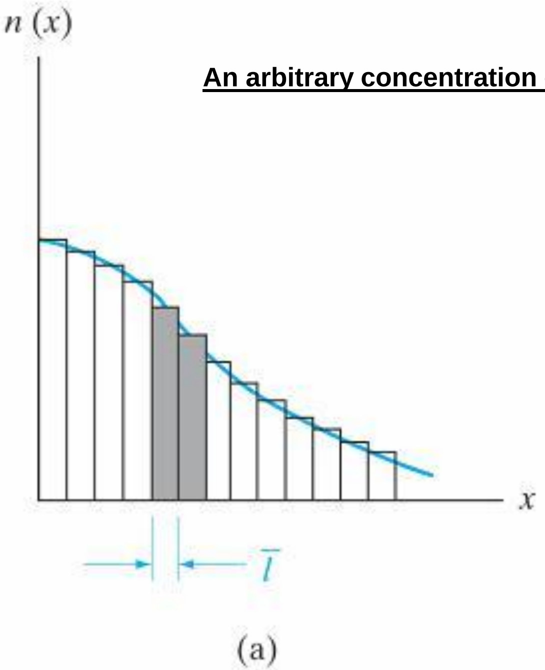


Figure 4.12

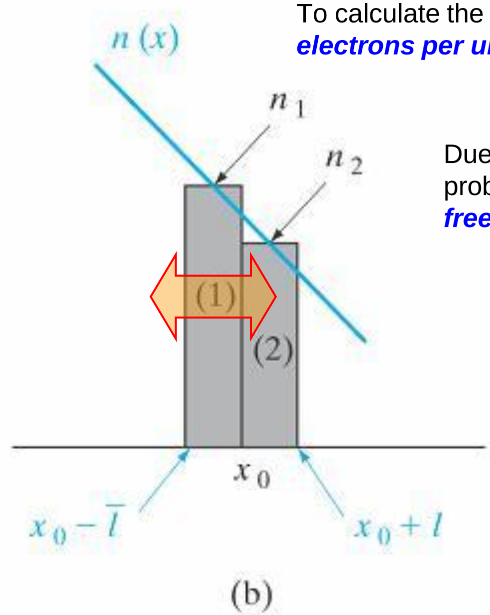
Spreading of a pulse of electrons by diffusion.

Let's consider an n-type semiconductor with a carrier concentration which varies in the x direction…

An arbitrary concentration gradient in 1D.



division of n(x) into segments of length equal to a mean free path for the electrons



expanded view of two of the segments centered at x_0.

To calculate the diffusion current, one must determine the ***net flow of electrons per unit time per unit area crossing the plane at x = 0.***

Due to their random motion, electrons at (1) have an equal probability of moving in either direction so ***within the mean free time one half of them will cross the plane at x = 0.***

The net number of electrons passing x_0 from left to right in one mean free time:

$$\frac{1}{2}(n_1\bar{l}A)-\frac{1}{2}(n_2\bar{l}A)$$

The rate of electron flow in the +x direction per unit area is given by:

$$\phi_n(x_0)=\frac{\bar{l}}{2\bar{t}}(n_1-n_2)$$

the former eq. can be written in terms of the carrier gradient dn(x)/dx:

$$\phi_n(x)=\frac{-\bar{l}^2}{2\bar{t}}\frac{dn(x)}{dx}$$

The quantity ($l^2/2t$) is called the electron diffusion coefficient D_n [cm^2/s].

Diffusion Current Density

Rate of electron flow in the +x direction

$$\phi_n(x) = -D_n \frac{dn(x)}{dx}$$

Rate of hole flow in the +x direction

$$\phi_p(x) = -D_p \frac{dp(x)}{dx}$$

A current density can flow in the absence of an electric field due to the ***diffusion of holes and electrons***, therefore:

$$J_{(diff)} = J_{n(diff)} + J_{p(diff)}$$

Current density is simply the product of the charge and particle flux, therefore:

$$J_{(diff)} = qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx}$$

Electrons and holes move together in a carrier gradient but the resulting currents are in opposite directions because of the opposite charge of electrons and holes.

Diffusion and Drift of Carriers

If an electric field is present in addition to the carrier gradient, the current density will each have a drift component and a diffusion component:

drift diffusion

$$J_n(x) = q\mu_n n(x)F(x) + qD_n \frac{dn(x)}{dx}$$

$$J_p(x) = q\mu_p p(x)F(x) - qD_p \frac{dp(x)}{dx}$$

The total current density is the sum of the contributions due to electrons and holes:

$$J(x) = J_n(x) + J_p(x)$$

This relation between particles flow and their currents can be better visualized:

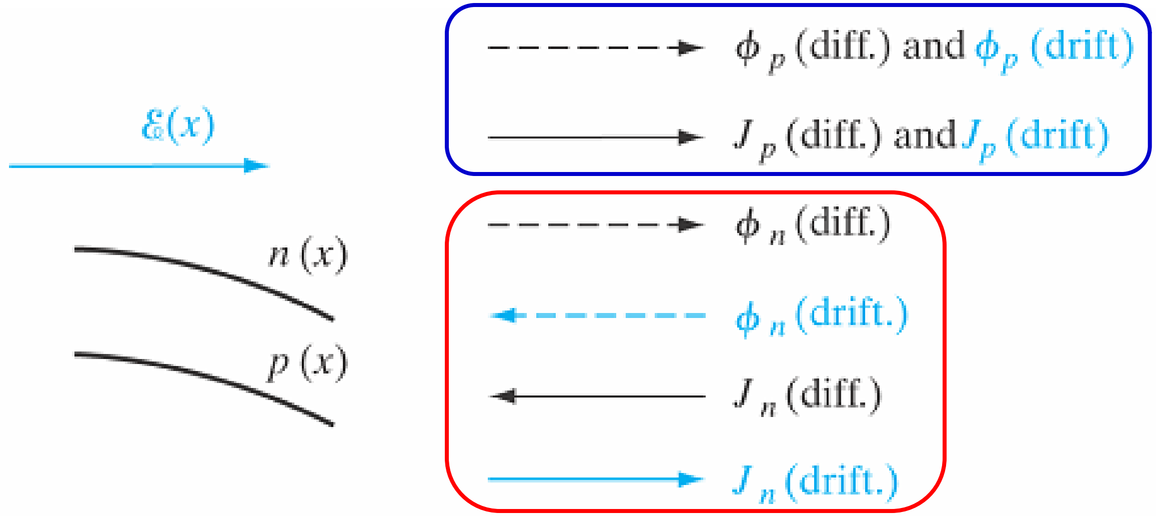


Figure 4.14

Drift and diffusion directions for electrons and holes in a carrier gradient and an electric field. Particle flow directions are indicated by dashed arrows, and the resulting currents are indicated by solid arrows.

The total current may be due primarily to the flow of electrons or holes, depending on the ***relative concentrations and the relative magnitudes and directions of electric field and carrier gradients***.

An important result is that the ***minority carries can contribute significantly to the current density through diffusion***. Since the drift terms are proportional to the carrier concentration, minority carrier seldom provide much drift current density. In the other hand, the diffusion current density is proportional to the gradient concentration.

In discussing the motion of carriers in an electric field, the influence of the field on the energies of electrons in the band diagrams take great importance.

Since electrons drift in a direction opposite to the field, the **potential energy for electrons increase in the direction of the field**. From the definition of electric field:

$$F(x) = -\frac{dV(x)}{dx}$$

We can relate $F(x)$ to the electron potential energy in the band diagram by choosing some reference in the band for electrostatic potential. Choosing E_i as a convenient reference, we can relate the electric field to this reference by:

$$F(x) = -\frac{dV(x)}{dx} = -\frac{d}{dx}\left[\frac{E_i}{(-q)}\right] = \frac{1}{q}\frac{dE_i}{dx}$$

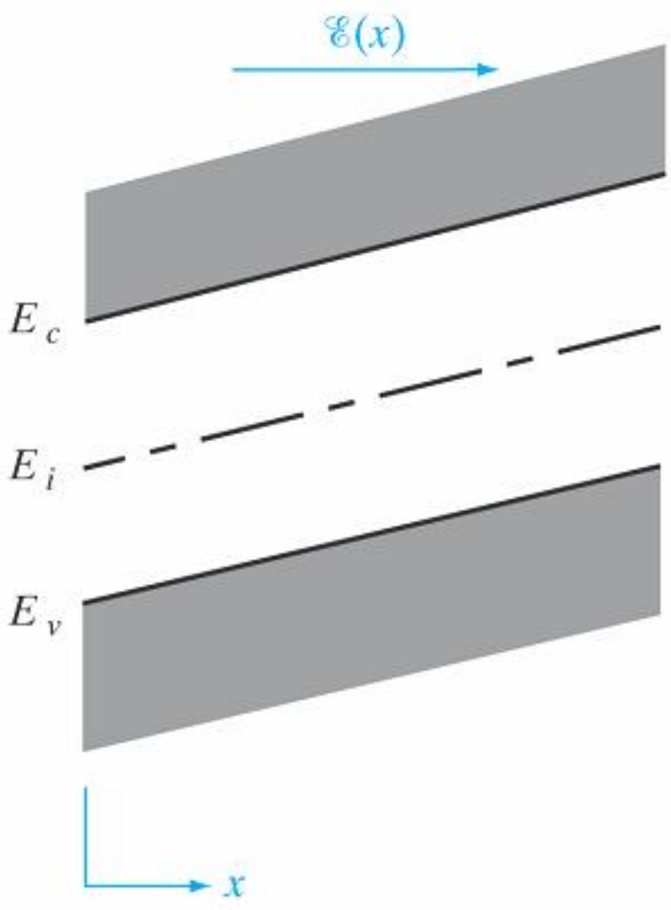


Figure 4.15

Energy band diagram of a semiconductor in an electric field $\mathscr{E}(x)$.

Einstein's Relationship

At equilibrium, no net current density flows in a semiconductor. Thus any fluctuation which would begin a diffusion current density also sets up an electric field which redistributes carriers by drift. ***An examination of the requirements for equilibrium indicates that the diffusion coefficient and mobility must be related***.

$$J_p = q\mu_p p(x) F(x) - qD_p \frac{dp(x)}{dx} = 0$$

$$F(x) = \frac{D_p}{\mu_p p(x)} \frac{dp(x)}{dx}$$

by using: $p(x) = n_i \exp\left[\frac{E_i - E_F}{kT}\right]$

$$F(x) = \frac{D_p}{\mu_p} \frac{1}{kT} \left[\frac{dE_i}{dx} - \frac{dE_F}{dx}\right]$$

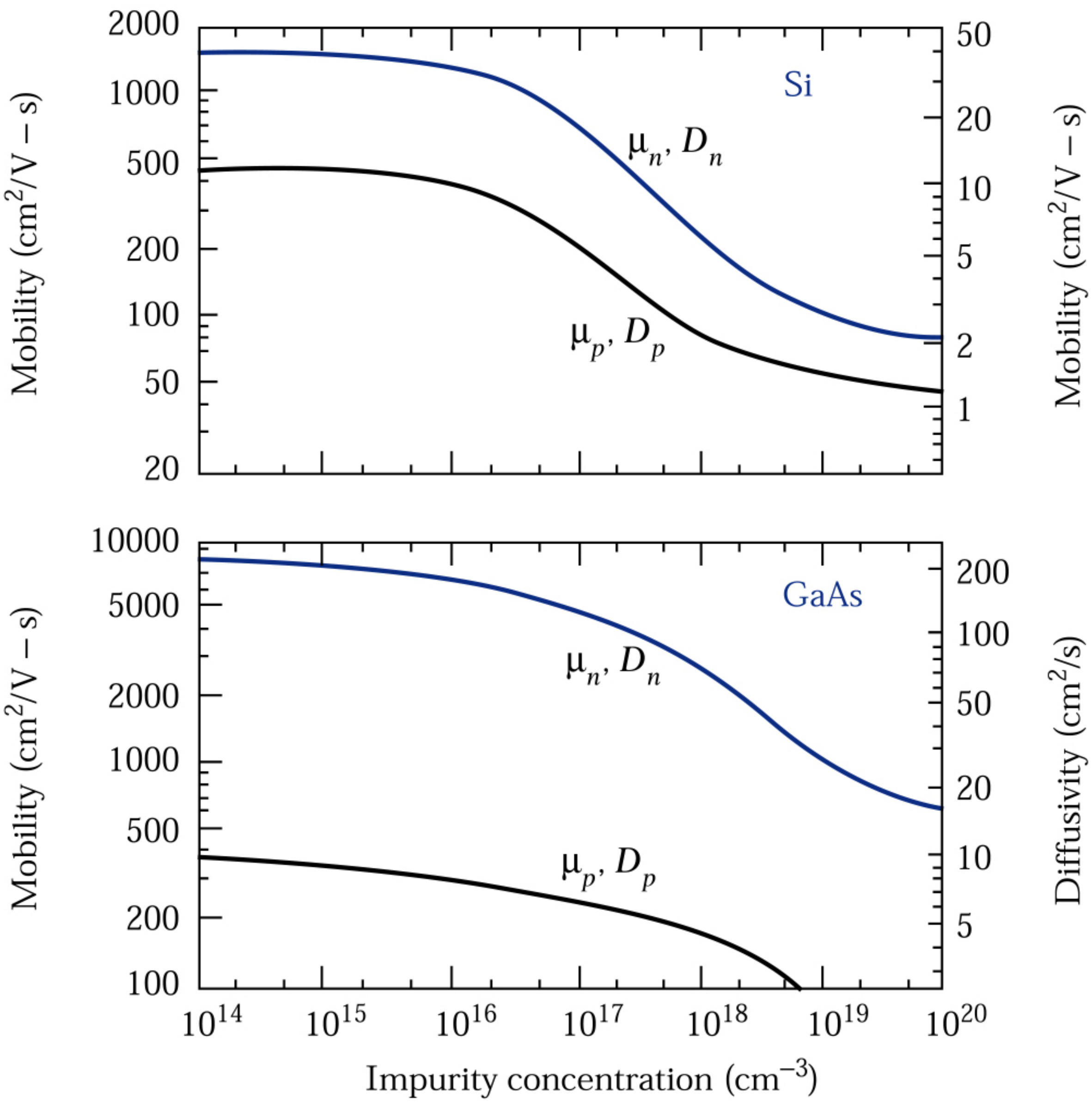
The equilibrium Fermi level does not vary with x, and the derivative of Ei is given by: $F(x) = \frac{1}{q} \frac{dE_i}{dx}$

Therefore:

$$\frac{D}{\mu} = \frac{kT}{q}$$

Einstein's relation

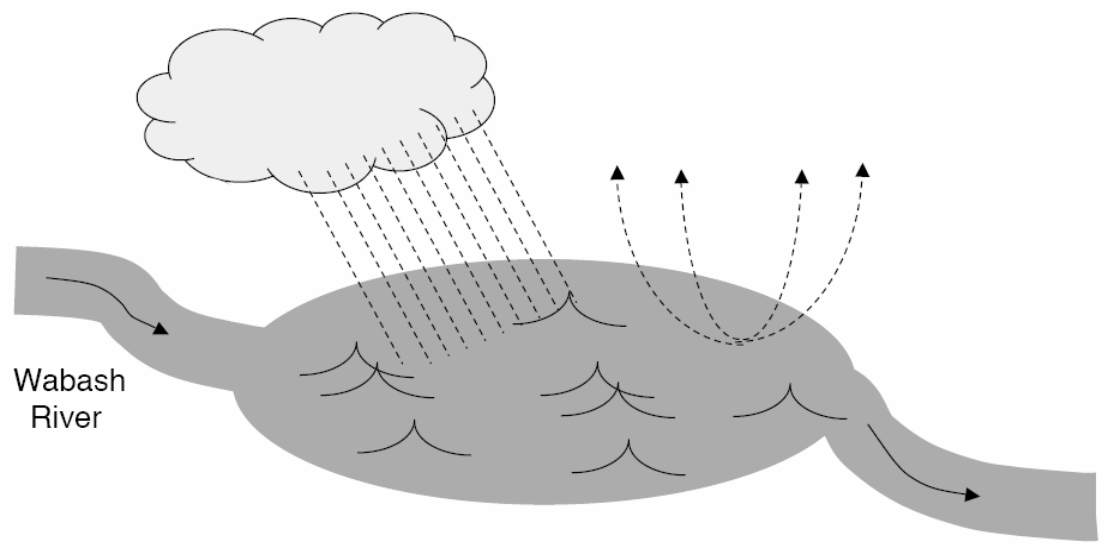
Mobilities and diffusivities in Si and GaAs at 300 K as a function of impurity concentration. (D / μ ≈ 26 mV @ 300 K).



Diffusion and Recombination: The Continuity Equation

A simple statement of conservation of particles emerges…

<u>**Rate of particle flow**</u>	=	Particle flow due to current	+	Particle gain due to generation	-	Particle loss due to recombination



Rate of increase of water level in lake = (in flow - outflow) + rain - evaporation

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\vec{J}_p / q\right) + G - R$$

NCN

Diffusion and Recombination: The Continuity Equation

For the discussion of excess carriers, we have thus far neglected the important ***effects of recombination***. These effects must be included in a description of conduction processes since recombination can cause a variation in the carrier distribution.

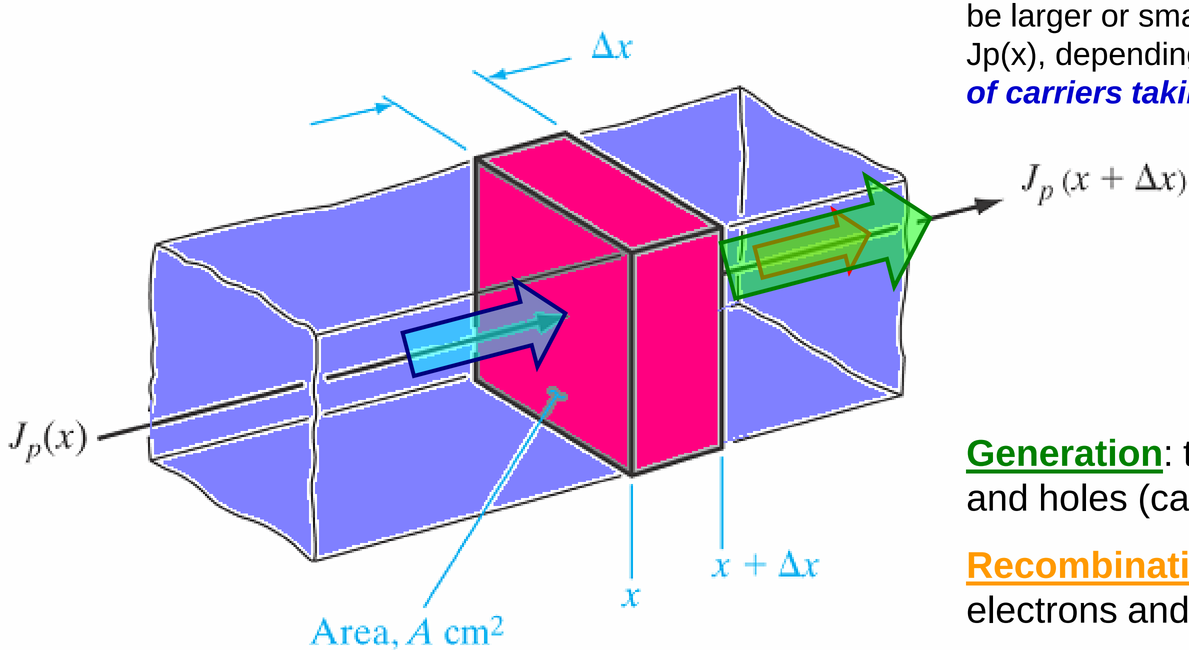


Figure 4.16

Current entering and leaving a volume ΔxA.

The hole current density leaving the volume, Jp(x+Δx), can be larger or smaller than the current density entering, Jp(x), depending on the ***generation and recombination of carriers taking place within the volume***.

Generation: the process whereby electrons and holes (carriers) are created.

Recombination: the process whereby electrons and holes (carriers) are annihilated.

Diffusion and Recombination: The Continuity Equation

The net increase in hole concentration per unit time, $\partial p/\partial t$, is the difference between the hole flux per unit volume entering and leaving, minus the recombination rate.

Hole flow rate into the slice at x is simply the current at x divided by the charge of an hole: $\frac{J_p(x)A}{q}$

Hole flow rate out of the slice at $x+dx$ is simply the current at $x+dx$ divided by the charge of an hole: $\frac{J_p(x+dx)A}{q}$

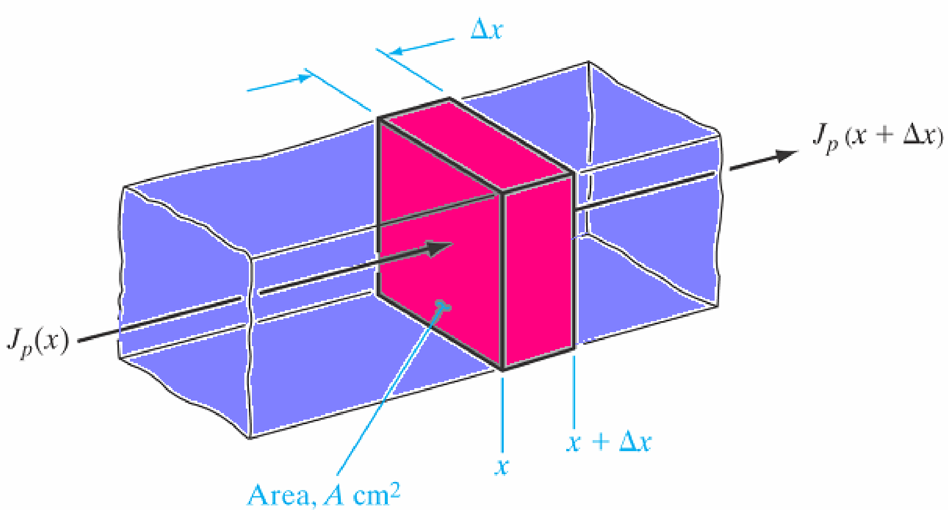


Figure 4.16

Current entering and leaving a volume $\Delta x A$.

The overall rate of change in the number of holes in the slice is:

$$\frac{\partial p}{\partial t}Adx = \left[\frac{J_p(x)A}{q} - \frac{J_p(x+dx)A}{-q}\right] + \left(G_p - R_p\right)Adx$$

Gp= hole generation rate. Rp= hole recombination rate.

The second term in the equation can be expanded into a Taylor series to:

$$J_p(x+dx) \cong J_p(x) + \frac{\partial J_p}{\partial x}dx$$

continuity equation for holes

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + \left(G_p - R_p\right)$$

continuity equation for electrons

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + \left(G_n - R_n\right)$$

Rate of Recombination

Electrons in Ec can recombine with holes in Ev and generate a photon. For a p-type semiconductor (p>>n), excess electrons injected by some means (e.g. the absorption of light) will recombine with the majority carriers (holes) with a ***recombination rate*** given by:

$$R_n = \frac{\Delta n}{\tau_n}$$

← Excess electron density

← Recombination lifetime

$\Delta n = n_p - n_{po}$ electron density – equilibrium electron density

τ_n mean time the electron is free before recombining with a hole.

Back to the continuity equation and introducing the recombination rate factor:

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + (G_p - R_p)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G_n - R_n)$$

$$\frac{\partial p_n}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G_p - \frac{p_n - p_{no}}{\tau_p}$$

$$\frac{\partial n_p}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - \frac{n_p - n_{po}}{\tau_n}$$

Diffusion and Recombination: The Continuity Equation

Things could start to get really complicated when we substitute the drift and diffusion currents in our earlier expressions … instead we will look at the special case where the ***current is carried only by the diffusion process and there is no generation***.

This is often the case when considering transport in p-n junction diodes and bipolar transistors when there are no optical excitations.

$$J_{n(diff)} = qD_n \frac{\partial n_p}{\partial x}$$

$$\frac{\partial n_p}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - \frac{n_p - n_{po}}{\tau_n} \Rightarrow \frac{\partial n_p}{\partial t} = D_n \frac{\partial^2 n_p}{\partial x^2} - \frac{n_p - n_{po}}{\tau_n}$$

In the steady state the time derivative is zero so:

$$D_n \frac{\partial^2 n_p}{\partial x^2} = \frac{n_p - n_{po}}{\tau_n} \Rightarrow \frac{\partial^2 n_p}{\partial x^2} = \frac{n_p - n_{po}}{D_n \tau_n} = \frac{n_p - n_{po}}{L_n^2}$$

Where we have defined an important quantity called the diffusion length,

$$L_n = \sqrt{D_n \tau_n} \qquad L_p = \sqrt{D_p \tau_p}$$

Ln/Lp is the average distance an electron/hole diffuses before recombining.

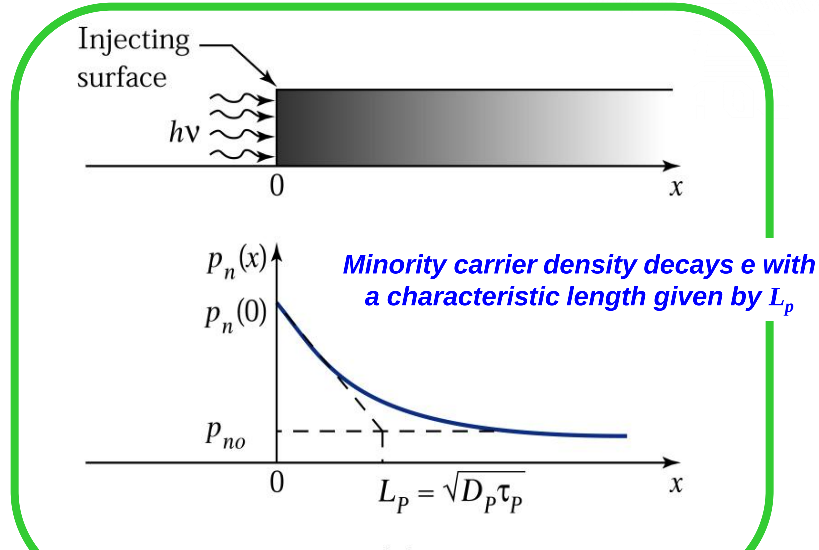
Diffusion Length

Consider an n-type semiconductor with steady state injection on one side

$W >> L_p$ →

This is the case for example in a long p-n diode where the carriers are injected at the origin and the excess density decays exponentially to zero deep within the bulk of the semiconductor.

$$p_n(x) = p_{no} + [p_n(0) - p_{no}]e^{-x/L_p}$$

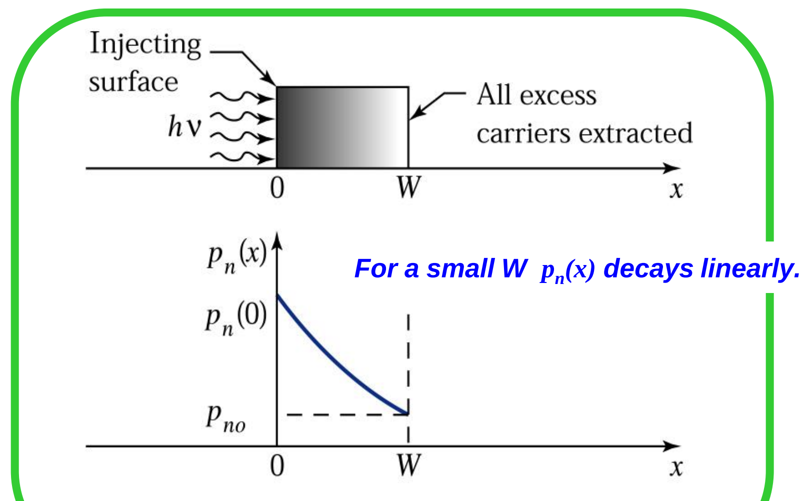


(a)

$W << L_p$ →

This is the case for example in a bipolar transistor with a narrow base region. In this case the carrier density varies essentially linearly from one boundary value to the other.

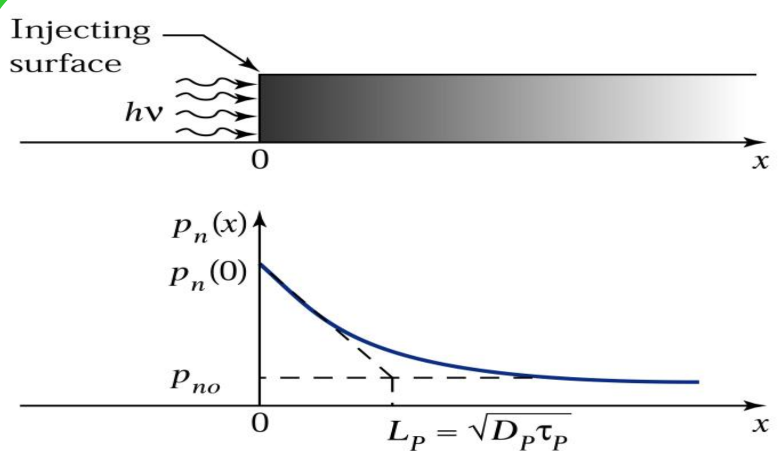
$$p_n(x) = p_{no} + [p_n(0) - p_{no}]\left[\frac{\sinh(W - x/L_p)}{\sinh(W/L_p)}\right]$$



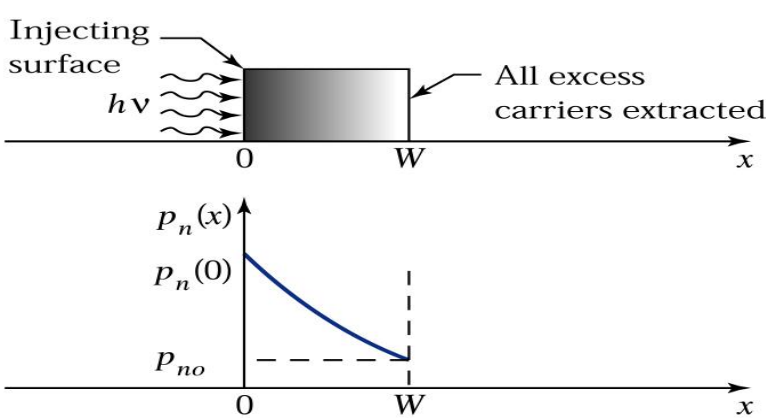
(b)

Diffusion Length

Consider an n-type semiconductor with steady state injection on one side



(a)



(b)

$$\frac{\partial p_n}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{no}}{\tau_p}$$

Boundary conditions are,

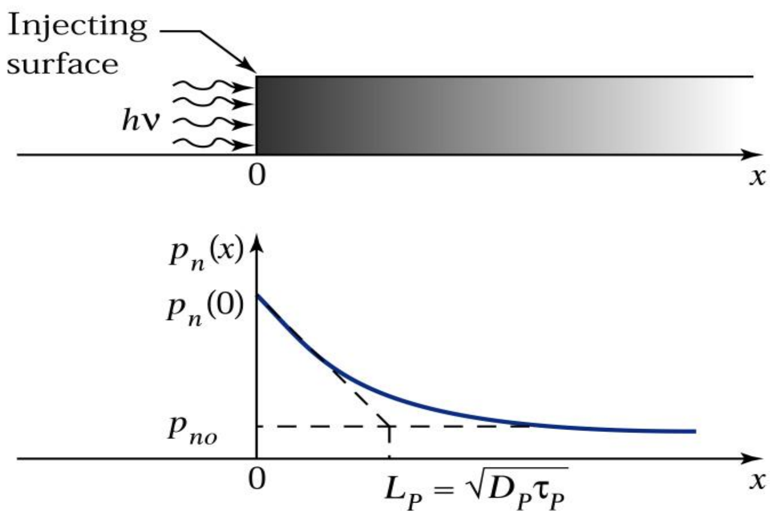
$$p_n(x=0) = p_n(0) \quad p_n(x \to \infty) = p_{no}$$

Solution of $p_n(x)$ is,

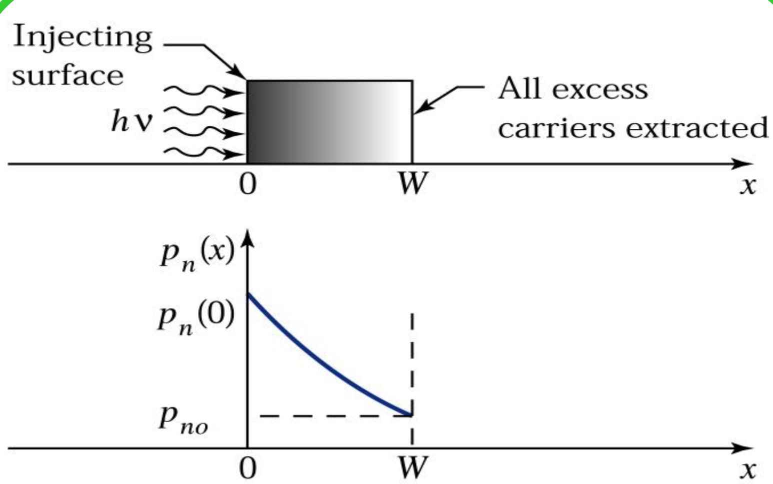
$$p_n(x) = p_{no} + [p_n(0) - p_{no}]e^{-x/L_p}$$

Minority carrier density decays with a characteristic length given by L_p

Semiconductor Devices, 2/E by S. M. Sze



(a)



(b)

Diffusion length

If all excess carriers are extracted at W (the thickness of the sample),

Boundary conditions are,

$$p_n(x=0) = p_n(0) \qquad p_n(W) = p_{no}$$

Solution of $p_n(x)$ is,

$$p_n(x) = p_{no} + [p_n(0) - p_{no}]\left[\frac{\sinh(W - x/L_p)}{\sinh(W/L_p)}\right]$$

For a small W $p_n(x)$ decays linearly

Homework

- Read section 4.4.5

  The Haynes-Shockley Experiment

- Read section 4.4.6

  Gradients in the Quasi-Fermi Levels