ON THE DESIGN OF A CLASS OF CHEBYSHEV FILTERS

David Báez-LópezJuan Manuel Ramírez, Luis Gerardo Guerrero, and Víctor Jiménez-Fernández
Department of Electronic Engineering
Universidad de las Américas, Puebla
Santa Catarina Martir, Cholula, Puebla, 72820
México

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ABSTRACT.- This work presents a modification to Chebyshev filter transfer functions to improve the group delay response. The technique consists in the shifting of the nearest to origin magnitude passband maximum, but taking special care in not modifying the normalized edge frequency ω_c. After making the proposed shifting in the transfer function, we will obtain improvements in the *group delay response.* Furthermore, the new frequency response will give us equally terminated *LC ladder* realization for even order functions.

1. INTRODUCTION

Chebyshev filter design is very popular because of a steeper roll-off at the band edge when compared with Butterworth filters. However, there are some disadvantages to this kind of approximating functions. First, the maximum passband gain response for even order Chebyshev filters happens at frequencies different from zero. This does not allow their *LC ladder* realization to be equally terminated, in other words the input and output resistors do not have the same value [1]. Odd order Chebyshev filter ladder realization has equal output and input resistors because their frequency response has a maximum at zero frequency. And second, Chebyshev filters phase variation depends upon the Chebyshev polynomial order, that is, the greater polynomial order corresponds to a worst phase response. This is seen in the group delay response of the transfer function. In this work we present a transformation that will alows us to improve the group delay response by making the filter to look as of lower order by shifting the lower maximun in the passband magnitude to zero.. In addition, we will allow even order passive realizations to have equal value terminating resistances. The proposed mapping can be applied repeatedely. The proposed mapping can also be applied to odd order functions.

2. TRANSFORMED CHEBYSHEV POLYNOMIALS

In order to find the modified Chebyshev function, we can express an even order Chebyshev polynomial as a binomial product series,

$$C_n(\omega) = \prod_{k=1}^{n/2} (\omega^2 - \omega_k{}^2) \qquad (1)$$

We must pay attention to the lowest minimum of $Cn(\omega)$ which will become the lowst maximum for the Chebyshev filter and which is given by

$$\omega_x = \cos\left[\frac{(n-1)\pi}{2n}\right] \qquad (2)$$

The proposed transformation that shifts the lowest maximum to the origin is then given by [3]

$$p = \sqrt{\frac{s^2 + \omega_x{}^2}{1 - \omega_x{}^2}} \qquad (3)$$

Where s denotes an original plane quantity, p denotes the new transformed p-plane quantity and ω_x is the nearest to the origin Chebyshev polynomial root. This transformation will be applied to each transfer function pole and to each reflection pole in the original transfer function.

This complex mapping can be applied *n/2-1* times until the original function (either odd or even order), which has *n/2* or *(n+1)/2* maximum is transformed into a function with one maximum as in a third order Chebyshev function. The recursive transformation is the same as in equation (3), but the new value of ω_x must be calculated after each mapping is applied.

3. EXAMPLE

As an example of the technique described above, let us consider a tenth order low pass filter with 1 dB of attenuation in the normalized passband, as shown in Fig. 1. After applying repeatedly the proposed mapping to this transfer function we obtain the magnitude characteristic of Fig. 2, which still looks like Chebyshev but of a reduced order, actually looks like a third order function. A comparison of the group delay for both transfer functionss is shown in Fig. 3 where we can see the improved delay response of the modified filter.

We can apply a synthesis procedure to the modified transfer functions following standard synthesis techniques [1,3,4]. This will render the elements of a passive *ladder realization* with equal terminating resistances.

It is possible to apply an extra-mapping to the last transfer function obtained in the procedure described above. This last application of the mapping shifts to the origin all of the the maxima . Although the mapping technique consists in the application one more time the equation (14), we have to consider this result separately because the magnitude-squared response obtained in this mapping loses its Chebyshev characteristic and looks like a Butterworth characteristic. We have called it , Modified-(n/2) or Planer Mapping and it is really important to pay attention to the surprisingly *Delay Response* improvement we get, because it is almost constant.

A sensitivity analysis on the original and transformed circuits shows that the modified circuits have better sensitivities as is shown in Fig. 7, where we show the function sensitivities of a sixth order filter, including a comparison with a lowpass Butterworth filter.

4. CONCLUSIONS

We have presented the application of a mapping to reduce the number of maxima in the passband ripple of Chebyshev filters. The main idea is to make a higher order Chebyshev filter to look like a smaller order filter in order to get a better delay response and to have an even order LC ladder realization with equal terminating resistances. We have obtained a modified Chebyshev filter with an improved time domain response and better sensitivity characteristics.

6. REFERENCES

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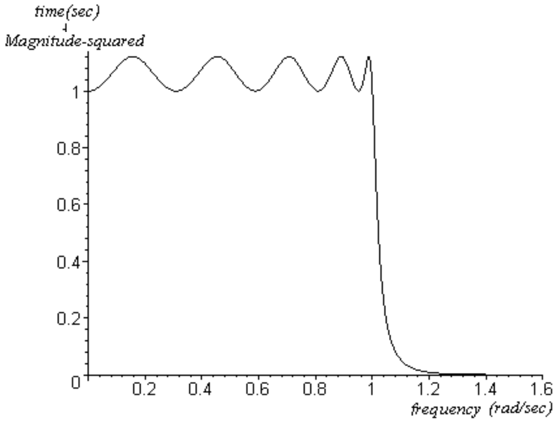

Fig. 1 Original tenth order magnitude characteristic.

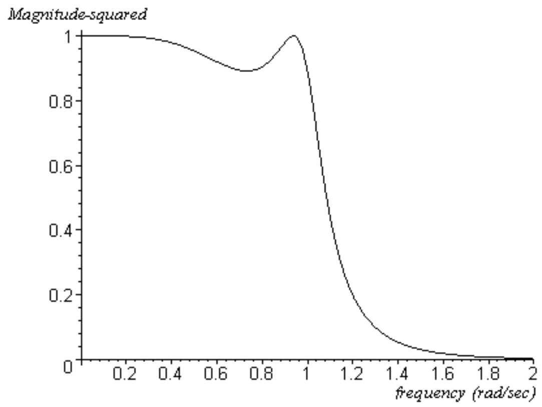

Fig. 2 Modified magnitude characteristic

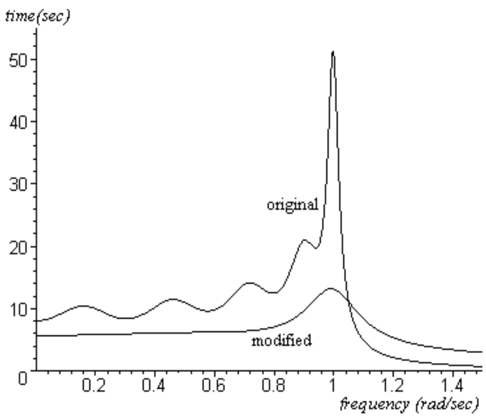

Fig. 3 Comparison between original and modified *group delay* , where we can see the improvement expected.

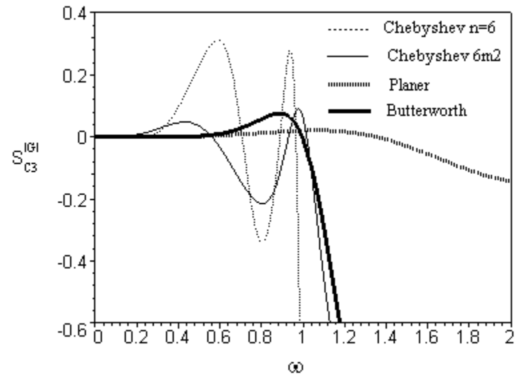

Fig. 4. Function sensitivity with respect to the third element C_3 in sixth order filters.