Wavelet-fuzzy logic approach to structural health monitoring

P. J. Escamilla-Ambrosio, *Member, IEEE*, X. Liu, N. A. J. Lieven, and J. M. Ramírez-Cortés, *Member, IEEE*

***Abstract*— In this work a novel wavelet-fuzzy logic approach to structural health monitoring is proposed based on wavelet transform theory and fuzzy logic technology. The proposed method combines the effectiveness of the Wavelet Packet Transform (WPT) as a tool for feature extraction and the capabilities of fuzzy sets to model vagueness and uncertainty. Two stages of operation are considered: pattern training and health monitoring. Pattern training is concerned with the determination of fuzzy sets based baseline patterns representing health condition states for which training data are available. Health monitoring is concerned with the classification of new data into the different structural health states. This classification problem is solved based on determining degrees of membership values to each one of the previously defined fuzzy patterns. In order to demonstrate the effectiveness and viability of the proposed approach, the method was applied to data collected from an experiment involving repeatedly impact excitations of an aluminum cantilever beam. Different damage cases in the beam where emulated by adding a lumped mass at different locations. The measured vibration response data provided by six accelerometers were analyzed. Results show that the method is effective in classifying the different damage cases.**

***Index Terms*— Damage classification, Structural health monitoring; Damage detection; Wavelet transform; Fuzzy logic.**

I. Introduction

Aerospace, civil and mechanical structures are subject to degradation due to the inherent ageing process, loads and environmental effects. Structural health refers to the structure ability to function, perform and maintain structural integrity throughout its entire lifetime [1]. Changes in a structure's health may affect its performance to a degree that it is not safe to be in operation and remedial maintenance actions are necessary. To ensure structural integrity and hence maintain safety, in-service health monitoring techniques are employed. When the monitoring system is able to interrogate sensor measurements autonomously for indications of structural damage, as a material, structural or functional failure, the system is referred to as a structural health monitoring (SHM) system [2]. The main attractive characteristic of implementing a SHM system arise from the number of benefits that can be obtained: reduced life cycle costs, reduced inspection/maintenance effort, improved performance, improved high rate operation availability, extended life of structures and improved safety.

Damage detection techniques, combined with advanced signal processing, are the core components of an SHM system. In this context, feature extraction is the process of identifying damage-sensitive information from measured data. A damage-sensitive feature is some quantity extracted from the measured system response data that is correlated with the presence of damage [3]. Ideally, extracted features will change in some consistent manner with increasing damage level or with different damage profiles. Ultimately, the goal is to distinguish a damaged structure accurately from an undamaged one based on the extracted features, a pattern classification (recognition) task. Two main alternative feature extraction methods have been proposed in the SHM literature, model based and waveform based [4]. The model based feature extraction methods consist of fitting some model, either physics based or data based, to the measured system response data. The parameters of these models or the predictive errors associated with these models then become the damage-sensitive features. Alternatively, the waveform based approaches extract features by directly comparing the sensor waveforms or spectra of these waveforms.

Due to the advantages of the Wavelet Transform (WT) over the Fourier Transform (FT) in terms of time-frequency resolution, the former technology has been gaining preference in the research community to explore waveform based feature extraction methods for SHM [5-6]. A possible drawback of the WT is that its resolution is rather poor in the high-frequency region. An extension of the WT, the Wavelet Packet Transform (WPT), uses redundant basis functions and hence can provide an arbitrary time-frequency resolution over all frequency regions [7-8]. Therefore, the WPT enables the extraction of features from signals that combine stationary and non-stationary characteristics even at higher frequencies.

In this work a novel waveform based feature extraction and damage classification technique for SHM is proposed. The novelty of the approach consists in combining the feature extraction and data compression capabilities of WPT [8] with

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the capability of fuzzy sets and fuzzy logic theory to deal with vagueness and uncertainty [9-10].

For the matter of space, the reader is refered to the background on WPT, fuzzy logic and fuzzy associative memory (FAM) to [8], [9] and [10], respectively. The remaining of the paper is organised as follows. In section II, the proposed wavelet-fuzzy logic approach to SHM is presented. The viability of the approach is denmostrated in section III by applying the method to vibration response data collected from an experiment involving repeatedly impact excitations of an aluminum cantilever beam. The conclusions of this work are given in section IV.

II. WPT-Fuzzy Logic SHM Method

A flowchart of the proposed WPT-fuzzy logic structural health monitoring method is shown in Fig. 1. Two stages of operation are considered: (i) baseline health pattern training and (ii) health monitoring. These stages of operation are explained in the following sections.

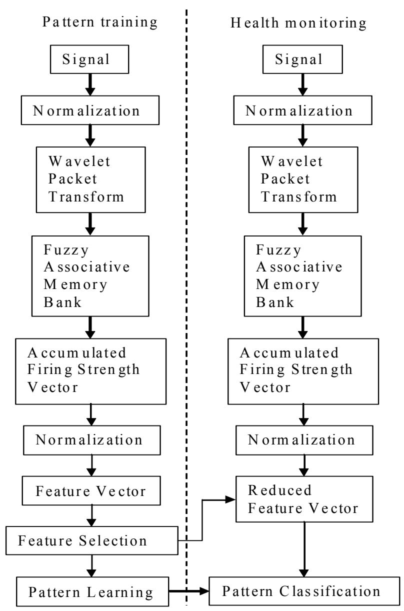

Fig. 1. Flow chart describing the stages of the proposed damage detection method.

A. Baseline health pattern training

In the first stage of operation the measured time-domain signal $f(t)$ is first normalized using the next relationship:

$$\hat{f}(t) = \left(\frac{f(t) - \mu_f}{\sigma_f} \right) \tag{1a}$$

$$\tilde{f}(t) = \left(\frac{\hat{f}(t)}{\max(abs(\hat{f}(t))} \right) \tag{1b}$$

where $\tilde{f}(t)$ is the normalized signal, and μ_f and σ_f are the mean and standard deviation of $f(t)$, respectively. Eq. (1) standardizes the signal $f(t)$ and puts its values between the range [-1,1].

After normalization, the resultant signal is wavelet packet decomposed into 2^n different frequency bands, with n being the level of decomposition. At level n, $2^n/2$ couples of low and high frequency components are obtained. The next stage in the proposed method is to input each pair of low and high frequency signals from level n into a two-dimension FAM. This means having a bank of $2^n/2$ identical FAMs processing these signals as can be appreciated in Fig. 2.

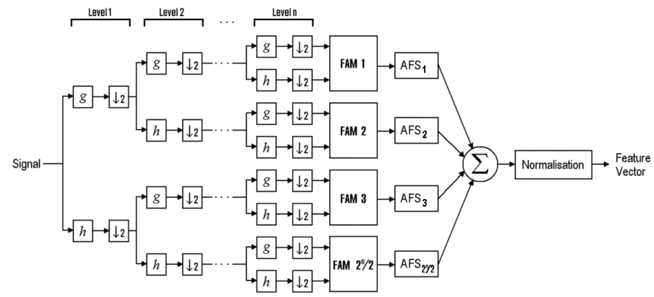

Fig. 2. WPT – FAM bank to perform the process of feature extraction.

The output of each FAM in the bank is a vector referred to as accumulated firing strength (*AFS*). This vector is calculated in the same manner in each FAM, in the following way. Assume that the original signal $f(t)$ is sampled and there are N_0 data points, then after normalization and after performing the WPT decomposition at level n, each of the resulting 2^n signals will have $N_0/2^n$ data points. Lets denote the couple of low (approximation) and high frequency (detail) signals being input to the lth FAM as $A^l_n(t)$ and $D^l_n(t)$, respectively; with the subscript n indicating the level of WPT decomposition and $l = 1,\ldots,2^n/2$. Thus, the approximation-detail data pair $(A^l_n(t), D^l_n(t))$, $t = 1,2,\ldots, N_0/2^n$ will have its firing strength or degree of membership to each one of the nR fuzzy regions on the FAM feature plane. The accumulated firing strength for the kth fuzzy region in the lth FAM can be written as:

$$\beta_k^{acc,l} = \sum_{t=1}^{N_0/2^n} \beta_k(t) = \sum_{t=1}^{N_0/2^n} (\mu_{A,i}(A^l_n(t)) \cdot \mu_{D,j}(D^l_n(t))) \tag{2}$$

After processing all data pairs $\{(A^l_n(t), D^l_n(t)),\ t = 1,2,\ldots, N_0/2^n\}$, the accumulated firing strengths for all the nR fuzzy regions on the feature plane are collected together to form the lth FAM output *AFS* vector. This is:

$$AFS_l = \begin{bmatrix} \beta_1^{acc,l} \\ \beta_2^{acc,l} \\ \vdots \\ \beta_{nR}^{acc,l} \end{bmatrix} = \begin{bmatrix} \sum_{t=1}^{N_0/2} (\mu_{A,1}(A^l_n(t)) \cdot \mu_{D,1}(D^l_n(t))) \\ \sum_{t=1}^{N_0/2} (\mu_{A,1}(A^l_n(t)) \cdot \mu_{D,2}(D^l_n(t))) \\ \vdots \\ \sum_{t=1}^{N_0/2} (\mu_{A,nA}(A^l_n(t)) \cdot \mu_{D,nD}(D^l_n(t))) \end{bmatrix}, \quad l = 1,\ldots,2^n/2$$

(3).

Once all the $2^n/2$ *AFS* vectors are available, a single *AFS* vector is obtained as:

$$AFS = AFS_1 + \cdots + AFS_{2^n/2} = \begin{bmatrix} \beta_1^{acc} \\ \beta_2^{acc} \\ \vdots \\ \beta_{nR}^{acc} \end{bmatrix} = \begin{bmatrix} \beta_1^{acc,1} + \cdots + \beta_1^{acc,2^n/2} \\ \beta_2^{acc,1} + \cdots + \beta_2^{acc,2^n/2} \\ \vdots \\ \beta_{nR}^{acc,1} + \cdots + \beta_{nR}^{acc,2^n/2} \end{bmatrix} \quad (4)$$

Finally, the *AFS* vector in (4) is normalized to a unit vector in order to minimize the sampling effect:

$$\overline{AFS} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{nR} \end{bmatrix} = \frac{1}{\sqrt{\sum_{k=1}^{nR} [\beta_k^{acc}]^2}} \begin{bmatrix} \beta_1^{acc} \\ \beta_2^{acc} \\ \vdots \\ \beta_{nR}^{acc} \end{bmatrix} \quad (5).$$

As each couple of low-high frequency signals correspond to different frequency bands and each couple is processed by identical FAMs, it is expected that the values in the *AFS* vectors, outputs of these FAMs, will be very dissimilar. For example, the highest values will be obtained from the FAMs processing the lower frequency bands, while the lowest values will be produced by the FAMs processing the higher frequency bands. However, by adding all the obtained *AFS* vectors to produce a single *AFS*, the information contained in each frequency band is preserved, although it is compressed when normalization to a unit length vector is performed.

The vector $\overline{AFS}$ defined by (5) is referred to as the feature vector and the process of obtaining it is referred to as feature extraction. Up to this point both baseline health pattern training and on-line monitoring share the feature extraction process. After that, in the case of baseline health pattern training, two stages need to be performed before pattern classification takes place. These stages are feature selection and pattern learning.

Feature selection is the process of selecting particular elements from the extracted feature vector in order to perform health state training. As a result of the feature extraction process a feature vector, $\overline{AFS}$, with *nR* elements is obtained. It is expected that some of the values in $\overline{AFS}$ will be more significant than others and the least of them will be dominating in magnitude. In that sense, the feature selection process consists in selecting or reducing the feature vector to contain only those values that are most significant in magnitude, this as part of the training process. Obviously, this reduced feature vector, referred to as $\overline{AFS}_R$, should contain the same elements for every of the different analyzed health states.

For the purposes of structural health state pattern learning, two assumptions are adopted in this study: (i) reliable healthy and damage time series vibration response signals are available; and (ii) these vibration response signals have resulted from repeatedly exciting the structure at a fixed location (e.g. cyclic impact loading). In order to perform pattern learning, the vibration response signals measured from the structure in the healthy and the different (levels of) damage conditions are divided into two sets, training and validation data. First, the training data are analyzed as indicated in the flow chart of Fig. 1 up to obtaining the corresponding reduced feature vectors. The level of WPT decomposition is determined through trial and error and observing the differences between the obtained feature vectors from healthy and damaged states. From this trial and error approach the reduced feature vector also is obtained.

The reduced feature vector is used to obtain a baseline or reference feature vector corresponding to each one of the healthy and damage states. A group of fuzzy membership functions that describe all health state conditions (or classes) are defined based on the statistics of each one of the elements in $\overline{AFS}_R$. First, assume that there are *n* time series vibration response signals available for training for each one of the structure health conditions. Therefore, after feature extraction, there will be *n* normalized accumulated firing strength vectors, denoted as $\overline{AFS}_1, \ldots, \overline{AFS}_n$, each one with *nR* elements denoted as $\phi_{i,1}, \ldots, \phi_{i,nR}$, $i = 1, \ldots, n$; and referred to as the *damage indices*. After feature selection, there will be *n* reduced accumulated firing strength vectors denoted as $\overline{AFS}_{1,R}, \ldots, \overline{AFS}_{n,R}$, each one with *m* damage indices denoted as $\phi_{i,1}, \ldots, \phi_{i,m}$, $i = 1, \ldots, n$. Now let us calculate the mean and standard deviation of each one of the damage indices in the reduced accumulated firing strength vectors, this is:

$$\bar{\phi}_j = \frac{1}{n} \sum_{i=1}^{n} \phi_{i,j} \quad (6)$$

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{n} \left(\phi_{i,j} - \bar{\phi}_j\right)^2}{n-1}} \quad \text{with:} \quad i = 1, \ldots, n; j = 1, \ldots, m \quad (7)$$

A set of patterns, in the form of fuzzy membership functions, representing each health state is defined using these statistics. Each health state or class is denoted as c_h, with $h = 1, .., nc$; where *nc* is the number of classes (health states). Having available the values of $\bar{\phi}_j$ and σ_j the membership function that describes the fuzzy set representing class c_h is defined by the following Gaussian function:

$$\mu_{c_h}^j = e^{\frac{-(x-\bar{\phi}_j)^2}{2(3\sigma_j)^2}} \quad (8)$$

where $\mu_{c_h}^j$ is the membership function of the fuzzy set representing the health state c_h based on the averaged observed index $\bar{\phi}_j$ and a spread of $3\sigma_j$. The spread $3\sigma_j$ of the fuzzy health pattern is selected based on ideas from SPC where usually the control limits are selected as ±3 the standard deviation of the feature being monitored. With this distribution, and assuming that the extracted damage indices of the new observed vibration response signals also have a Gaussian probability distribution, it is expected that most of the new signals will be correctly classified. Therefore, there will be *m* fuzzy sets all of them associated to class c_h. The process of obtaining these membership functions is represented in Fig. 3. Once all *m* fuzzy sets for all the *nc* classes (health states) have been determined, the process of pattern learning is terminated. This means that a total of $m \times nc$ fuzzy sets would have been defined in this way (see Fig. 4).

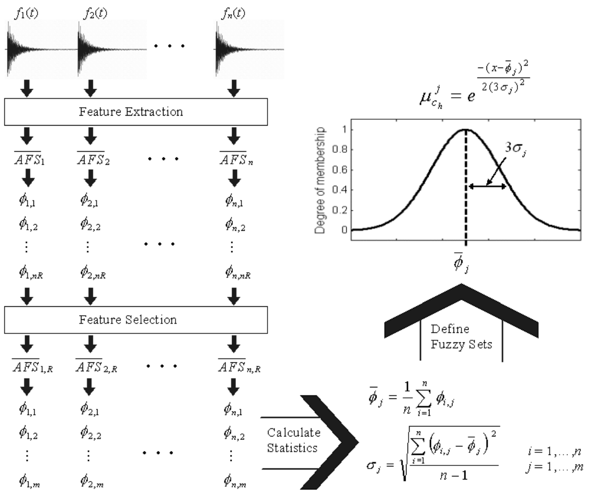

Fig. 3. Feature selection and pattern determination process.

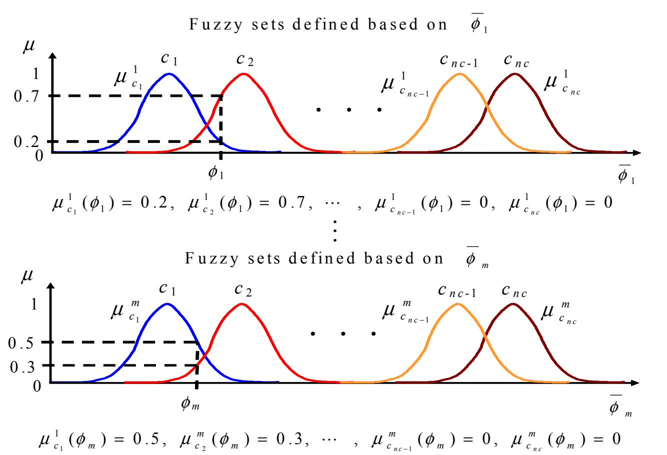

Fig. 4. Determination of the degree of membership to each health state class (fuzzy set).

B. Health monitoring

The operational stage of health monitoring involves the determination of the state of health of the structure under analysis based on the defined baseline fuzzy sets health patterns. Therefore, when a new time series vibration response signal is available, the state of health of the structure will be determined based on the degree of membership of the extracted damage indices in each one of the baseline fuzzy membership patterns. To do this, all the stages from normalization, up to obtaining the reduced feature vector, are performed (as can be seen in Fig. 1). In this case, the obtained reduced feature vector is denoted as $\overline{AFS}_R$. The individual values of $\overline{AFS}_R$, which are the damage indices, are denoted as $\phi_1,\ldots,\phi_m$. The damage indices are used to perform pattern recognition in the following manner. Each baseline health state class is characterized by a fuzzy set. Therefore, each index ϕ_j will be member of each one of the baseline health state classes (defined based on the corresponding averaged observed index $\bar{\phi}_j$) in some degree, as is represented graphically in Fig. 4. These degrees of membership can be grouped in a matrix μ:

$$\mu = \begin{bmatrix} \mu^1_{c_1}(\phi_1) & \mu^1_{c_2}(\phi_1) & \cdots & \mu^1_{c_{nc}}(\phi_1) \\ \vdots & \vdots & \vdots & \vdots \\ \mu^m_{c_1}(\phi_m) & \mu^m_{c_2}(\phi_m) & \cdots & \mu^m_{c_{nc}}(\phi_m) \end{bmatrix} \quad (9)$$

where $\mu^j_{c_h}(\phi_j)$ can be read as "the degree in which damage index ϕ_j is member of class c_h"; the superscript j in $\mu^j_{c_h}$ is used to indicate that the corresponding membership function has been defined based on $\bar{\phi}_j$ and σ_j. Each column in the matrix μ can be interpreted as a fuzzy rule, and all of them can be considered as a fuzzy rule base. This is:

Rule 1: If ϕ_1 is c_1 and ϕ_2 is c_1 and … and ϕ_m is c_1, then $f(t)$ belongs to class c_1 in a degree μ_{c_1}

Rule 2: If ϕ_1 is c_2 and ϕ_2 is c_2 and … and ϕ_m is c_2, then $f(t)$ belongs to class c_2 in a degree μ_{c_2}

⋮

Rule nc: If ϕ_1 is c_{nc} and ϕ_2 is c_{nc} and … and ϕ_m is c_{nc}, then $f(t)$ belongs to class c_{nc} in a degree $\mu_{c_{nc}}$

where μ_{c_h} is the combined degree of membership to class c_h, $h = 1,\ldots,nc$. Denoting the fuzzy intersection with the operator $\wedge$, μ_{c_h} is calculated by evaluating the antecedent part of each rule as:

$$\mu_{c_h} = \wedge \mu^j_{c_h}(\phi_j) \quad (10)$$

Adopting the algebraic product operator as the fuzzy intersection, (10) is transformed to:

$$\mu_{c_h} = \prod_{j=1}^{m} \mu^j_{c_h}(\phi_j) = \mu^1_{c_h}(\phi_1)\cdot\mu^2_{c_h}(\phi_2)\cdot\ldots\cdot\mu^m_{c_h}(\phi_m);\ h = 1,\ldots,nc \quad (11)$$

Hence, a single degree of membership to each one of the nc classes or health states is obtained using (11). These degrees of memberships can be grouped in a class vector result C:

$$C = [\mu_{c_1} \quad \mu_{c_2} \quad \cdots \quad \mu_{c_{nc}}]^T \quad (12)$$

Finally, the health state of the system under analysis is determined by selecting the index of the maximum value in vector C as:

$$Class = \text{Index max (C)} \quad (13).$$

III. Method Applied to Experimental data

In this section the results of an experimental study involving pulse-excited vibration tests of an aluminum cantilever beam carried out in the laboratory are presented. The beam is 900mm length and cross section 25.45 × 6.47mm, see Fig. 5. Six accelerometers (7g each) were screwed to the bottom surface along the centerline at selected positions (sensor 1 at 150mm, sensor 2 at 300mm, sensor 3 at 450mm, sensor 4 at 600mm, sensor 5 at 750mm, and sensor 6 at 895mm from the fixed point, respectively) to pick up the acceleration at different positions. Consistent pulse loads were emulated by using a rubber coated steel ball with a diameter of 22mm and a mass of 32g. The ball was released and free fall through a metallic guide from a distance of 20cm with the guide tilted 25° as is shown in Fig. 9. The ball hit a region roughly 5mm away from the free end of the beam and then bounced off the beam creating a pulse load on the beam. Data sets were collected at a sampling rate of 10KHz and last for 4 seconds. Different damage cases were emulated by adding a lumped mass (22g) at different locations in the beam. In damage 1 (D1) the mass was screwed above sensor 1, in damage 2 (D2) the mass was screwed above sensor 2, and so on until damage 5 (D5) where the mass was screwed above sensor 5 (no lumped mass was added above sensor 6). The healthy state (H) corresponds to the case where no mass is added to the beam. Table 1 summarizes the different damage cases.

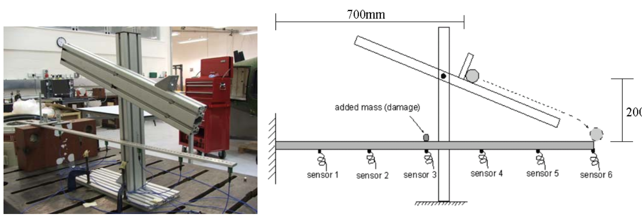

Fig. 5. Cantilever beam.

Table 1
Summary of damage cases

Damage Case	Location of Damage	Damage Description
Healthy (H)	-	No lumped mass is added
Damage 1 (D1)	150mm	Adding a lumped mass of 22g
Damage 2 (D2)	300mm	Adding a lumped mass of 22g
Damage 3 (D3)	450mm	Adding a lumped mass of 22g
Damage 4 (D4)	600mm	Adding a lumped mass of 22g
Damage 5 (D5)	750mm	Adding a lumped mass of 22g

A total of 10 pulse-excited vibration tests were performed for every health state (five damage cases and the healthy state), and the corresponding vibration response signals collected from every sensor were recorded. Thus, a total of 60 time series for every sensor were available for analysis. Although the method could be applied to analyze the data obtained from a single sensor, which may be enough to detect the different damage cases, it was decided to apply the method to the data obtained from each sensor independently. This was done with the aim of determining which sensor location produced the best damage discrimination results.

Thus, the proposed damage detection method was applied to the recorded data from each sensor. For the first stage of operation, which obtains the baseline health patterns, the first 5 time series signals, referred here as training signals, from every health state were used. For the second stage of operation, which performs health monitoring (pattern classification), the remaining 5 time series signals from every health state were used in order to validate the approach.

Simulations of the method were developed in the MATLAB/Simulink environment. After normalization, the vibration time series signals were wavelet packet transformed up to level 6 using the Haar wavelet. Then, 32 identical two-dimensions FAMs processed each couple approximation-detail signals. The FAM was designed using six and four triangular fuzzy sets as linguistic variables for the approximation and detail signals, respectively. The number of fuzzy sets and their distribution on the respective universe of discourse were determined empirically. The universe of discourse for each sensor case was determined based on the maximum range of the decomposed signal corresponding to the lowest frequency band. It was observed that better results were obtained when the number of fuzzy sets defined for the approximation signal (A) is greater than the number of fuzzy sets defined for the detail signal (D).

Therefore, once the training signals from every sensor had been wavelet packet transformed and processed by the FAMs, the obtained AFS vectors were added and the resultant vector normalized to unit length to obtain the reference feature vector $\overline{AFS}$. Each feature vector contains $6 \times 4 = 24$ damage index elements corresponding to 24 fuzzy regions or fuzzy rules. During the feature selection stage it was determined that the elements corresponding to the regions (rules) 10, 11, 14 and 15 were the most significant. These rules are expressed as:

R10: If (x_A is NS and x_D is NS), then region R_{10} is activated
R11: If (x_A is NS and x_D is PS), then region R_{11} is activated
R14: If (x_A is PS and x_D is NS), then region R_{14} is activated
R15: If (x_A is PS and x_D is PS), then region R_{15} is activated

Therefore, the corresponding damage indices were selected to form the reduced feature vector $\overline{AFS}_R$:

$$\overline{AFS}_R = \begin{bmatrix} \phi_{10} \\ \phi_{11} \\ \phi_{14} \\ \phi_{15} \end{bmatrix} = \frac{1}{\sqrt{\sum_{k=1}^{24} [\beta_k^{acc}]^2}} \begin{bmatrix} \beta_{10}^{acc} \\ \beta_{11}^{acc} \\ \beta_{14}^{acc} \\ \beta_{15}^{acc} \end{bmatrix} \tag{14}$$

After obtaining the reduced feature vectors for each one of the training signals, their statistics were used to determine the baseline health pattern as a group of fuzzy sets. Examples of the fuzzy set patterns obtained for each one of the health class states are shown in Fig. 6 for the case of time response signals provided by sensor 4. Once the baseline fuzzy sets patterns have been obtained, the operation stage of health monitoring can be initiated. Therefore, the method was applied to the 30 time series signals left for testing purposes, 5 testing signals for every health state case. After performing feature extraction, the state of health of the structure was determined based on the degree of membership of the extracted damage indices in each one of the baseline fuzzy membership patterns. The next rule base was applied and evaluated:

Rule 1: If ϕ_{10} is H and ϕ_{11} is H and ϕ_{14} is H and ϕ_{15} is H, then $f(t)$ belongs to class H in a degree μ_H
Rule 2: If ϕ_{10} is D1 and ϕ_{11} is D1 and ϕ_{14} is D1 and ϕ_{15} is D1, then $f(t)$ belongs to class D1 in a degree μ_{D1}
Rule 3: If ϕ_{10} is D2 and ϕ_{11} is D2 and ϕ_{14} is D2 and ϕ_{15} is D2, then $f(t)$ belongs to class D2 in a degree μ_{D2}
Rule 4: If ϕ_{10} is D3 and ϕ_{11} is D3 and ϕ_{14} is D3 and ϕ_{15} is D3, then $f(t)$ belongs to class D3 in a degree μ_{D3}
Rule 5: If ϕ_{10} is D4 and ϕ_{11} is D4 and ϕ_{14} is D4 and ϕ_{15} is D4, then $f(t)$ belongs to class D4 in a degree μ_{D4}
Rule 6: If ϕ_{10} is D5 and ϕ_{11} is D5 and ϕ_{14} is D5 and ϕ_{15} is D5, then $f(t)$ belongs to class D5 in a degree μ_{D5}

where μ_i, i = H, D1, D2, D3, D4, D5, is calculated by applying (11) and the health state is determined by applying (13). Note that the rule base and class determination is evaluated 6 times, once for each sensor, using the corresponding baseline fuzzy health patterns (fuzzy sets) defined during the stage of pattern training. The health state classification results obtained for the signals provided by sensor 4 are shown in Fig. 7, where the left plot indicates the combined degree of membership of each one of the health state classes μ_i, while the right plot shows the resultant class index (the index of the maximum degree of membership to each fuzzy health pattern). Note that the indices mean: 1 = H, 2 = D1, 3 = D2, 4 = D3, 5 = D4 and 6 = D5, respectively.

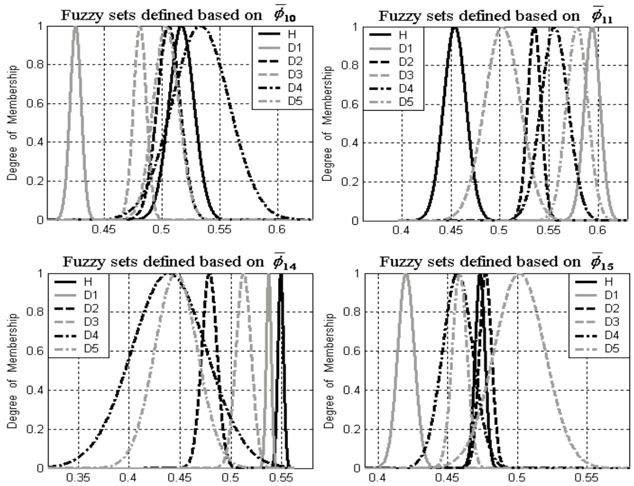

Fig. 6. Fuzzy health patterns (fuzzy sets) obtained from the statistics of the damage indices 10, 11, 14 and 15 obtained based on signals from sensor 4.

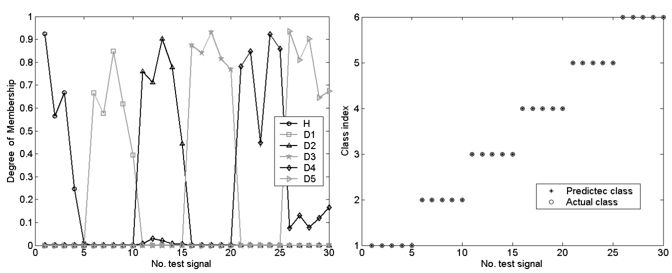

Fig. 7. Health state classification results using data measurements provided by sensor 4.

From the results obtained for all the sensors, although not presented here for a matter of space, several observations can be made: 1) If the purpose is to discriminate between a healthy and a damaged structure (usually known as level 1 damage assessment), then the method is 100% effective: the healthy state is correctly classified in all cases regardless of the sensor providing the data. 2) If the purpose is not only to detect damage but also to locate the damage, then the method is 100% successful when using the data measurements from sensors 3, 4, and 5. 3) The best results are obtained with the data measurements provided by sensor 4, which location can be selected as the best sensor location in this study. This was inferred from the corresponding degrees of membership to each one of the class state patterns. Note in Fig. 7 that the maximum degree of membership values correspond to the correct class and the remaining degree of membership values are driven by the method to very low values, reducing the possibility of misclassification. 4) The effectiveness of the method using data measurements from sensors 1, 2 and 6 are 83.33%, 80% and 86.67%, respectively. Therefore, the worst sensor location in this study is that of sensor 2. 5) The damage D2 is the hardest to detect using the proposed method. 6) Note that only five training signals from each health state were needed to obtain the baseline fuzzy class patterns and once the baseline fuzzy class patterns are available, the method is suitable to perform continuous on-line SHM.

IV. Conclusions

In this work a wavelet packet transform (WPT)-fuzzy logic (FL) theory approach to damage detection and location has been presented. The method combines the effectiveness of WPT as a tool for feature extraction and the capabilities of fuzzy sets to model vagueness and uncertainty. The method was applied to data measurements corresponding to the vibration response of a cantilevered beam with different damage locations. The data provided from six sensors was analyzed independently. The method was 100% effective in differentiating the healthy state from any damage states regardless of the sensor providing the data. For the task of damage localization, the method effectively located all damage cases when using the data measurements from sensors 3, 4 and 5, independently. When using the data from sensors 1, 2 and 6, the method was effective in locating the damage cases in 80% or more. The method can be used to determine the best sensor location based on the classification results. For the example presented, the best sensor location is that of sensor 4. Future work includes the implementation of the method in a digital signal processor (DSP) to perform SHM of different engineered structures.

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