On Digital Signal Processing Understanding through Simulation and Animation Tools

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Abstract - **This paper describes the use of simulation and animation tools based on MathCAD, aimed to support the understanding of basic principles of digital filters. An important feature is the interaction student-computer to simulate different problems with parameter changes and its instantaneous evaluation by the software, which motivates the reasoning and understanding of the mathematical concepts by the student. In addition, the animation options in MathCAD allow the students to create virtual environments which resemble the real instrumentation procedures in the laboratory, such as the typical frequency sweep based on a signal generator and the oscilloscope. A collection of different study cases in the area of digital signal processing such as filter design, Nyquist criterium, digital filtering in the time domain by difference equations, relation to frequency response in both, continuous and discrete domains, and modulation, is described.**

Index Terms – Digital signal processing, digital filters, simulation, animation, frequency response.

I. INTRODUCTION

In the last years there has been a strong development of simulation software to be used in almost every field of engineering. In the specific case of digital signal processing, there is a great variety of professional software as well as scholar work, which can be extremely useful in the design and testing of digital circuits and systems, or in the simulation and analysis of digitized signals [1-4]. However, the fundamental concepts involved in the theory of signal processing can be understood in a simple and efficient way when the student is directly involved with the design, writing, and testing of the corresponding mathematical model through visualization tools and animation features. The interactivity with the computer motivates the student to experiment with the change of several parameters, encouraging exploration and generation of hypothesis in a dynamic way. The software package MathCAD from Math Soft, is a suitable and affordable tool to support theoretical discussion as well as lab work in the topic of digital filters taught at a senior undergraduate course [5]. As a rapid simulation tool, this package enables the student to enter and execute complex mathematical models without writing any code, motivating the understanding of the topic. A special feature of MathCAD is the creation of animation videos in *avi* format, which allows the student to experiment with changes in some parameter, and create applications that resemble the operation of instruments in the laboratory, such as a sweep generator, an oscilloscope, or a spectrum analyzer [6,7].

II. PROPOSED EXPERIMENTS

A collection of experiments in the form of MathCAD worksheets directly related to the sylabus of a junior undergraduate course on Digital Signal Processing is proposed. The list of the experiments is:
- Sampled signals and Nyquist criterium
- Convolution; time and frequency domains
- FIR filters by implementing difference equations
- IIR filters by implementing difference equations
- Filter design; Butterworth, Chebyshev and Elliptic approximations
- Relation between poles and zeroes in the unit circle and the frequency response
- Window-based design of digital filters
- Backward, forward, and bilinear transformations.

In each experiment, a MathCAD worksheet is to be developed, exploring the effect of parameter changes in the results obtained, and implementing video animations as analytical tools, that in some cases emulate the results obtained using laboratory instruments.

III. AN EXAMPLE ON DIGITAL FILTERS SIMULATION

As an example, let us review the mathematical fundaments of digital filters, with comments about specific difficulties commonly found from the perspective of a junior level undergraduate student. A digital filter can be regarded as the physical implementation of a difference equation applied to a discrete signal in real time. Although there are many books and references in the topic [8,9], it is common in a first approach, to experience difficulties in visualizing the relation between a difference equation and the frequency response of the corresponding transfer function. The understanding of this concept is supported with the aids of simulation and animation tools developed in MathCAD. Figure 1 shows a 4th order digital

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filter implemented as the cascade structure of two second-order sections. There are several methods that can be used to obtain the filter coefficients starting from a set of specifications, such as: Frequency transformations from analog low-pass filter prototypes, poles and zeros mapping from the s-plane to the z-plane, and least-squares design methods [8]. Let us consider the 4th order band-pass elliptic transfer function given by (1) with the coefficients indicated in table [I].
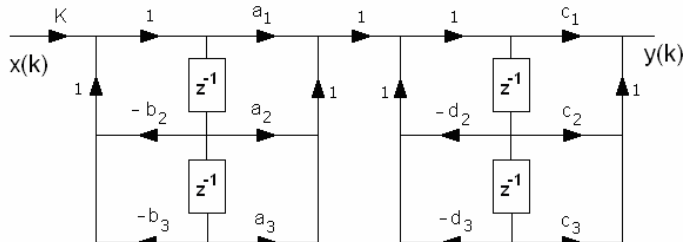

Fig. 1. 4th order digital filter in cascade form of biquadratic sections.

$$H(z) = K\left(\frac{a_1 + a_2 z^{-1} + a_3 z^{-2}}{1 + b_2 z^{-1} + b_3 z^{-2}}\right)\left(\frac{c_1 + c_2 z^{-1} + c_3 z^{-2}}{1 + d_2 z^{-1} + d_3 z^{-2}}\right).$$

$$(1)$$

TABLE I
COEFFICIENTS FOR A 4TH ORDER BAND-PASS ELLIPTIC TRANSFER FUNCTION

1st section coefficients	2nd section coefficients
a_1 = 2.1483116	c_1 = 0.778657
a_2 = 1.732215	c_2 = -1.359387
a_3 = 2.148311	c_3 = 0.778357
b_2 = -0.644891	d_2 = -0.990263
b_3 = 0.919515	d_3 = 0.926438

Applying a factorization in two terms and the z-transform, the output is obtained from the difference equations given by (3), where u(k) is an auxiliary variable.

$$H(z) = \frac{Y(z)}{X(z)}$$
$$= H_1(z)H_2(z)$$
$$= \frac{Y(z)}{U(z)}\frac{U(z)}{X(z)},$$

$$(2)$$

$$u[k] = a_1 x[k] + a_2 x[k-1] + a_3 x[k-2]$$
$$- b_2 u[k-1] - b_3 u[k-2]$$
$$y[k] = K\{c_1 x[k] + c_2 x[k-1] + c_3 x[k-2]\}$$
$$- d_2 u[k-1] - d_3 u[k-2].$$

$$(3)$$

These equations can be implemented in MathCAD using one-dimensional arrays in vector form. In order to verify graphically the filter response, the transfer function is evaluated using sinusoidal steady state concepts. Let us consider a discrete

sinusoidal input x(t) represented in phasor form as $\tilde{x}(t)$. The output is obtained from the convolution of the signal with the filter impulse response as:

$$\tilde{x}(k) = e^{j\omega k}$$

$$(4)$$

$$\tilde{y}(k) = \tilde{x}(k) * h(k)$$

$$(5)$$

$$\tilde{y}(k) = \sum_{m=-\infty}^{\infty} h(m)e^{j(k-m)\omega}$$
$$= \sum_{m=-\infty}^{\infty} h(m)e^{jk\omega}e^{-jm\omega}$$

$$(6)$$

$$= e^{jk\omega} \sum_{m=-\infty}^{\infty} h(m)e^{-jm\omega} .$$

The discrete time Fourier transform of the impulse response corresponds to the transfer function of the digital filter as:

$$H(e^{jw}) = \sum_{m=-\infty}^{\infty} h(m)e^{-jm\omega} .$$

$$(7)$$

Substituting in (6), and taking the real part, the output signal in the time domain is given as:

$$y(k) = \mathrm{Re}\{\tilde{y}(k)\}$$
$$y(k) = |H(e^{j\omega})|\cos[\omega k + \arg H(e^{j\omega})].$$

$$(8)$$

Next, a worksheet with the simulation of these equations is presented in order to illustrate the behavior of the system when a sinusoidal input is applied, and a frequency sweep between 0 and Fs/2 is performed. The requirements for the construction of the corresponding worksheet are given in Table II.

TABLE II
WORK SPECIFICATIONS FOR A DIGITAL FILTER

1) Use a fixed number of points to define in a one-dimensional array a sinusoidal input signal.
2) Input the coefficients corresponding to the difference equations of a fourth order elliptic band-pass digital filter in cascade form.
3) Input the difference equations.
4) Plot the sinusoidal input and output signals from the evaluation of the difference equations.
5) Input and plot the corresponding transfer function in the z-domain. Locate in this plot the frequency of the input signal.
6) Using the concept of steady state response, evaluate the magnitude and phase of the transfer function in the value of the frequency corresponding to the sinusoidal input signal. Plot input and output signal in order to compare them with the signals obtained from difference equations.
7) Generate a video animation in *avi* format, which resembles a sweep function generator and an oscilloscope to visualize the

input and output signals, and a spectrum analyzer in order to locate the corresponding frequency in each point.

A. MathCAD worksheet

Figure 2 shows the worksheet generated following the work specifications described in table I.
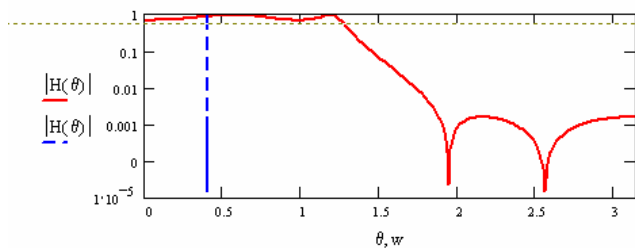
FOURTH ORDER LOW PASS ELLIPTIC DIGITAL FILTER - IE422 - 2007

SAMPLING FREQUENCY: $Fs := 20000$ INPUT SIGNAL FREQUENCY: $f := 1300$

NUMBER OF CYCLES: $\qquad n := \dfrac{f}{Fs} \cdot 64$

SAMPLING TIME: $\Delta T := \dfrac{\pi}{32}$

NORMALIZED FREQUENCY: $w := n \cdot \Delta T$
DATA RANGE: $k := 0 .. 63$

SINUSOIDAL INPUT SIGNAL: $x_k := \cos(w \cdot k)$

COEFFICIENTS: $\qquad K := 0.0016$

$a1 := 2.214139 \quad a2 := 1.634267 \quad a3 := 2.214139 \quad b2 := -1.23055 \quad b3 := 0.563459$

$c1 := 7.720792 \quad c2 := 12.929621 \quad c3 := 7.72079 \quad d2 := -0.64697 \quad d3 := 0.864992$

INITIALIZATION:

$x_{64} := 0 \quad x_{65} := 0 \quad u_k := 0 \quad y_k := 0$

DIFFERENCE EQUATIONS:

$u_{k+2} := a1 \cdot x_{k+2} + a2 \cdot x_{k+1} + a3 \cdot x_k - b2 \cdot u_{k+1} - b3 \cdot u_k$

$y_{k+2} := K \cdot \left(c1 \cdot u_{k+2} + c2 \cdot u_{k+1} + c3 \cdot u_k\right) - d2 \cdot y_{k+1} - d3 \cdot y_k$

PLOT OF INPUT AND OUTPUT SIGNALS FROM DIFFERENCE EQUATIONS:
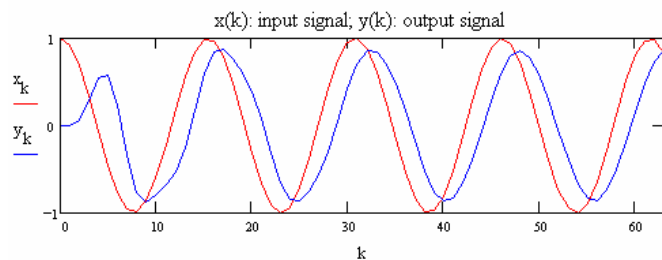
EVALUATION OF THE TRANSFER FUNCTION:

$\theta := 0, 0.01 .. \pi \qquad z(\theta) := e^{i \cdot \theta}$

$H(\theta) := K \cdot \dfrac{a1 + a2 \cdot z(\theta)^{-1} + a3 \cdot z(\theta)^{-2}}{1 + b2 \cdot z(\theta)^{-1} + b3 \cdot z(\theta)^{-2}} \cdot \dfrac{c1 + c2 \cdot z(\theta)^{-1} + c3 \cdot z(\theta)^{-2}}{1 + d2 \cdot z(\theta)^{-1} + d3 \cdot z(\theta)^{-2}}$

$w = 0.408 \qquad |H(w)| = 0.865 \qquad \arg(H(w)) = -0.754$

OUTPUT SIGNAL FROM SINUSOIDAL STEADY STATE RESPONSE:

$x(t) := \cos(w \cdot t)$

$t := 0, \Delta T .. 63$

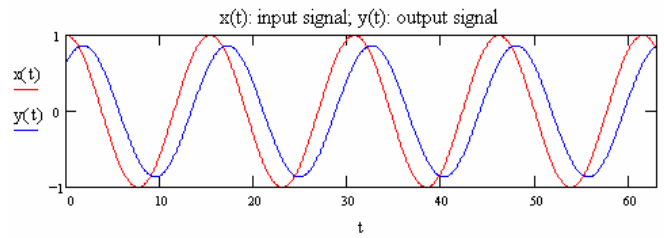$y(t) := |H(w)| \cdot \cos(w \cdot t + \arg(H(w)))$

Fig. 2. MathCAD worksheet of a low-pass elliptic digital filter.

B. Digital filter animation

A very interesting feature of MathCAD is the option to implement animations from a worksheet allowing a parameter to change in some predefined range, and giving to the student an additional insight about the behavior of the model. In the analysis of digital filters, for instance, a frequency sweep can be easily implemented by the creation of a video file in *avi* format, which resemble a virtual instrument supported by the corresponding mathematical model. Figure 3 shows a snapshot of the video file obtained from the worksheet previously described. By sliding the button in the player, the student would be able to locate graphically and numerically the frequency of interest, obtaining the input and output waveforms, as well as the magnitude and phase from the filter frequency response. This feature, in addition to the MathCAD signal processing extension pack functions, makes it a very valuable educational tool in this field, and in engineering education in general.
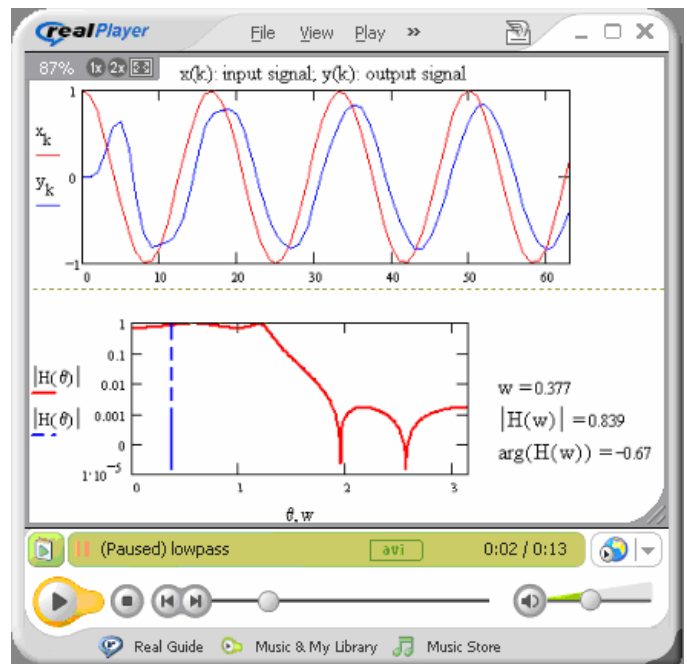
Fig. 3. Animation of a low-pass elliptic digital filter.

IV. AN EXAMPLE ON SAMPLED SIGNALS AND NYQUIST CRITERIUM.

A basic concept in discrete signal analysis is the sampling process and the Nyquist criterium, as well as the associated aliasing phenomena. A sampled signal can be represented by the multiplication of the continuous signal with an impulse train, as given by (9).

$$x(nT) = x_i(t) \sum_{n=-\infty}^{\infty} \delta(t - nT) \tag{9}$$

$$\Im\{x(nT)\} = \frac{1}{2\pi} \Im\{x_i(t)\} * \Im\left\{\sum \delta(t - nT)\right\}$$

$$= \frac{1}{2\pi} X(\omega) * \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta\left(\omega - n\frac{2\pi}{T}\right)$$

$$= \frac{1}{T} \sum_{n=-\infty}^{\infty} X\left(\omega - n\frac{2\pi}{T}\right) \qquad . \tag{10}$$

If ω_m is the bandwith of the original signal, then the condition $\dfrac{2\pi}{T} \geq 2\omega_n$ must be satisfied in order to avoid an overlapping of the spectrum due to aliasing. This concept can be easily animated in MathCAD, sampling a sinusoidal function, and exploring the aliasing resulted from undersampling, as described in table III.

TABLE III
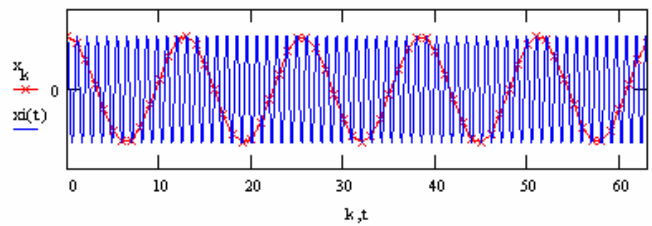WORK SPECIFICATIONS FOR THE ALIASING
EXPERIMENT

1)   Define and plot a continuous sinusoidal signal.
2)   Define and plot a discrete sinusoidal signal which represents a sampled version of the original signal.
3)   Propose a fixed sampling frequency. Allow the frequency of the continuous signal to vary from 0 to the sampling frequency.
4)   Obtain and plot the Fourier Transform of the discrete signal.
5)   Generate a video animation in *avi* format, in such a way that the signal frequency can be modified with a slide control.

A. MathCAD worksheet

Figure 4 shows the worksheet generated following the work specifications described in table III.

B. Aliasing animation

A video file in avi format is created using the MathCAD resources. This animation resembles the signals obtained in the laboratory using an oscilloscope, a signal generator, and a spectrum analyzer. The aliasing is observed when the signal frequency goes beyond the Nyquist frequency Fs/2. Figure 5 shows a snapshot of the video file obtained from the worksheet described in Figure 4. By sliding the button in the player, the student would be able to increase the frequency of the sinusoidal input signal in order to analyze the effects of undersampling in the time and frequency domains.
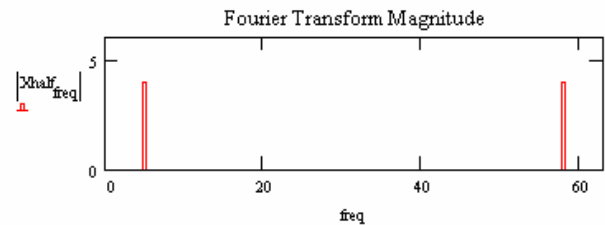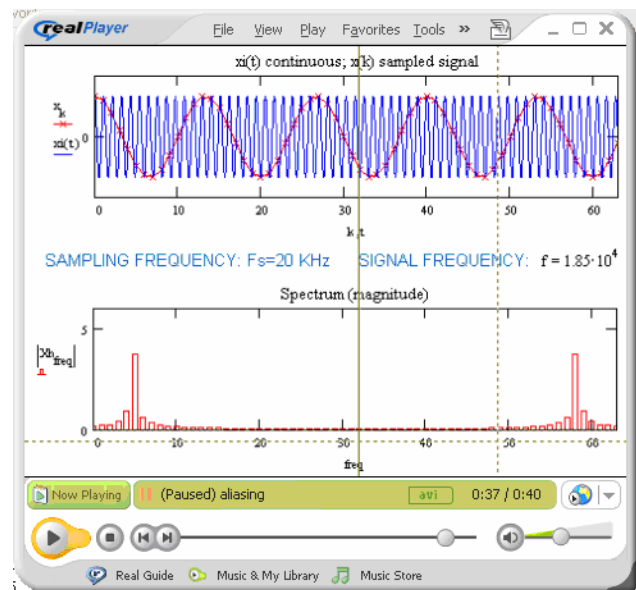

Fig. 4. MathCAD worksheet of sinusoidal sampled signal.

Fig. 5. Animation of a sinusoidal sampled signal.

V. CONCLUSION

The use of simulation and animation tools using MathCAD to support experimental lab work in the junior undergraduate course of Digital Signal Processing was presented. Animation helps to visualize many mathematical concepts that are difficult to explain verbally or to understand from static pictures. The video files create a virtual laboratory environment giving an additional insight to the students about the mathematical concepts behind the models, encouraging exploration and generation of hypothesis in a dynamic way. MathCAD is a powerful tool in the engineering field to simulate different systems, and it was found particularly useful as a tool to support experimental lab work in signal analysis. One of the most important features is the interactivity between the student and the computer to simulate different problems with parameter changes and an instant evaluation by the software, which motivates the reasoning and understanding of the material. Concepts such as filtering by difference equations, aliasing due to undersampling, and others are visualized in an easy way through the use of the techniques described. The animations described are currently used in the graduate course on digital signal processing at the National Institute of Astrophysics, Optics, and Electronics, with a very good acceptance by the students as a learning support. Additional material with the same scope is continuously developed by the students and the instructor. Further asset on the learning improvement using the described material is currently in progress.

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