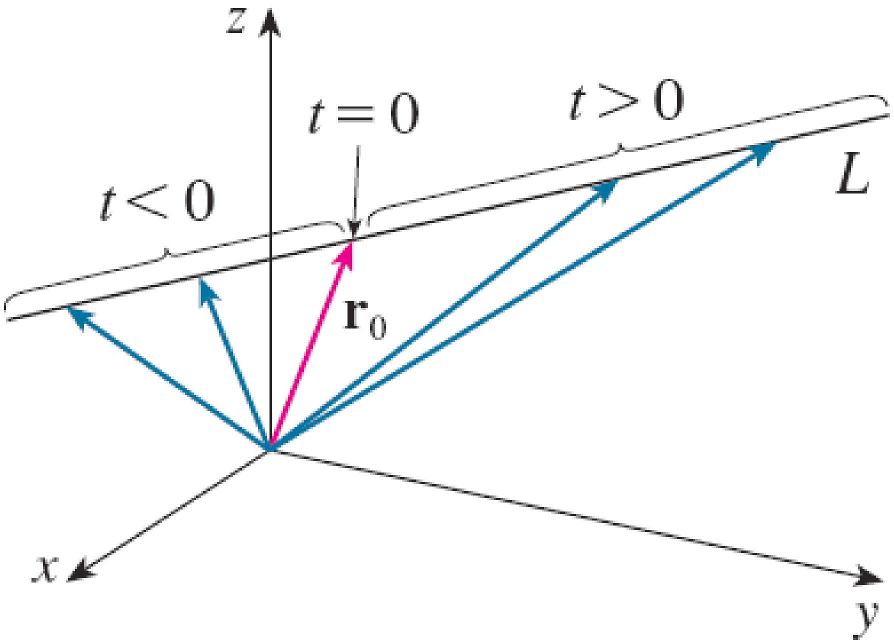
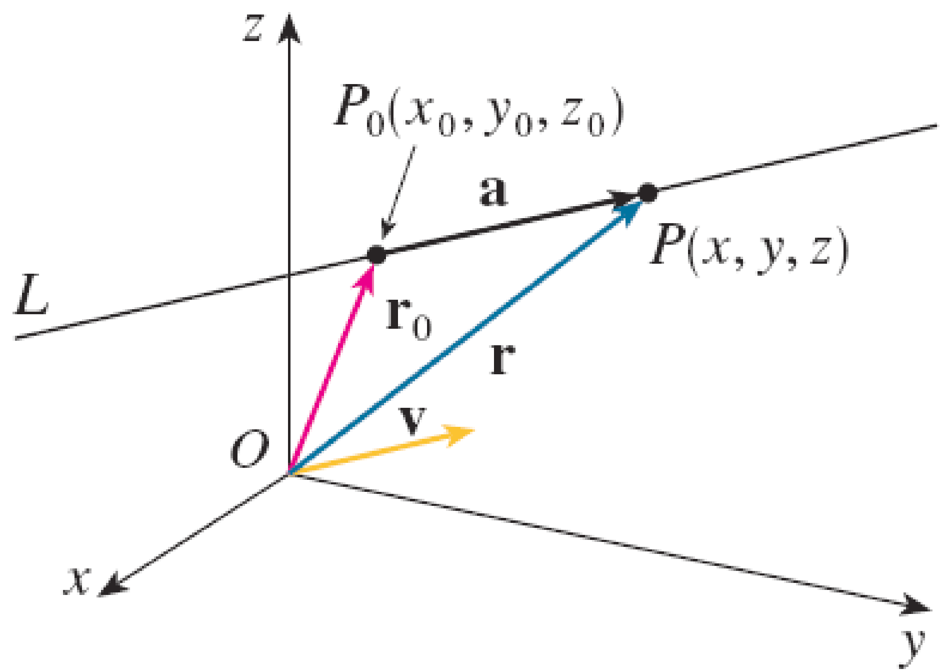
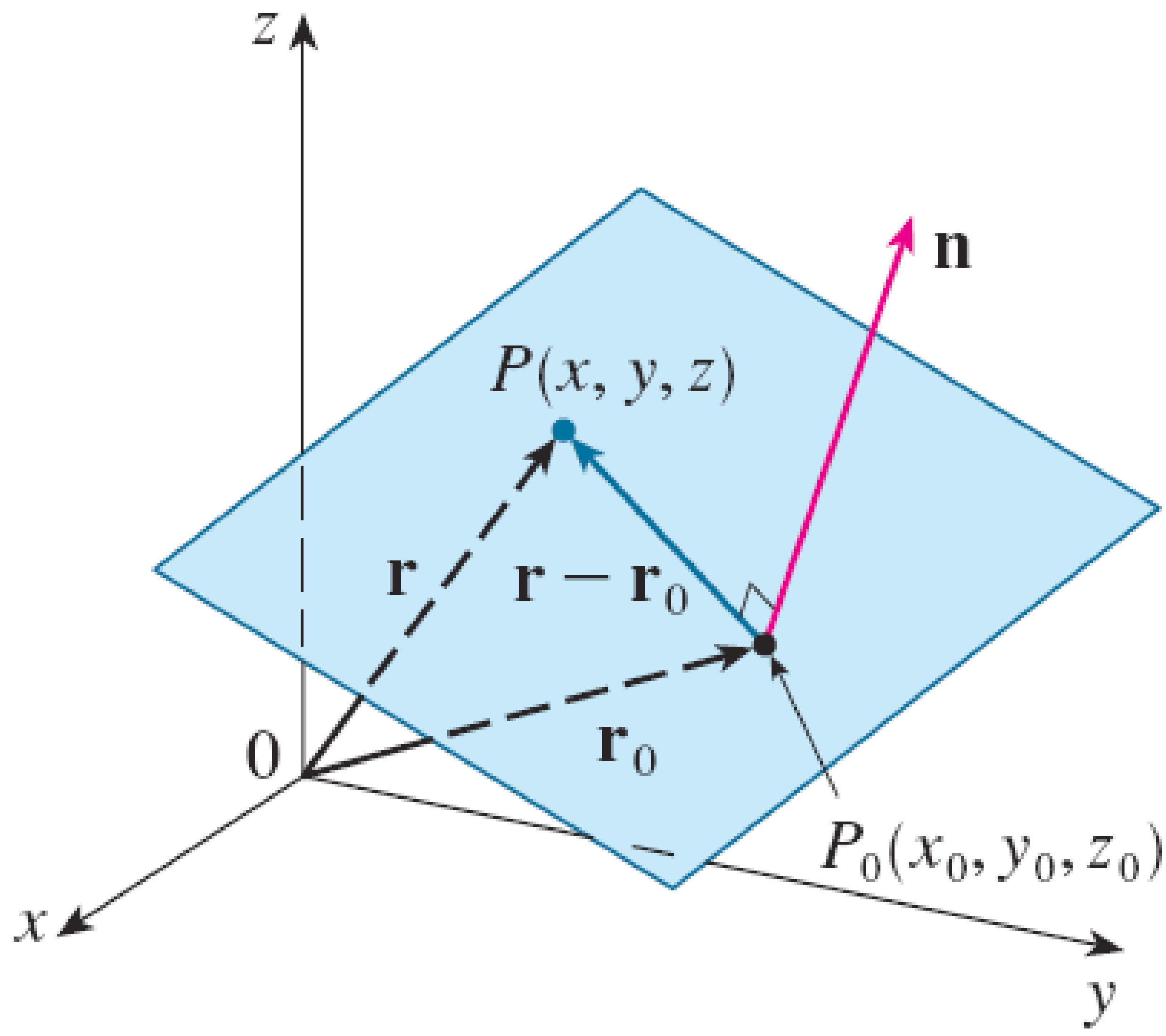
Superficies y derivadas

Dr. Rogerio


z
t = 0
t > 0
t < 0
L
r0
x
y

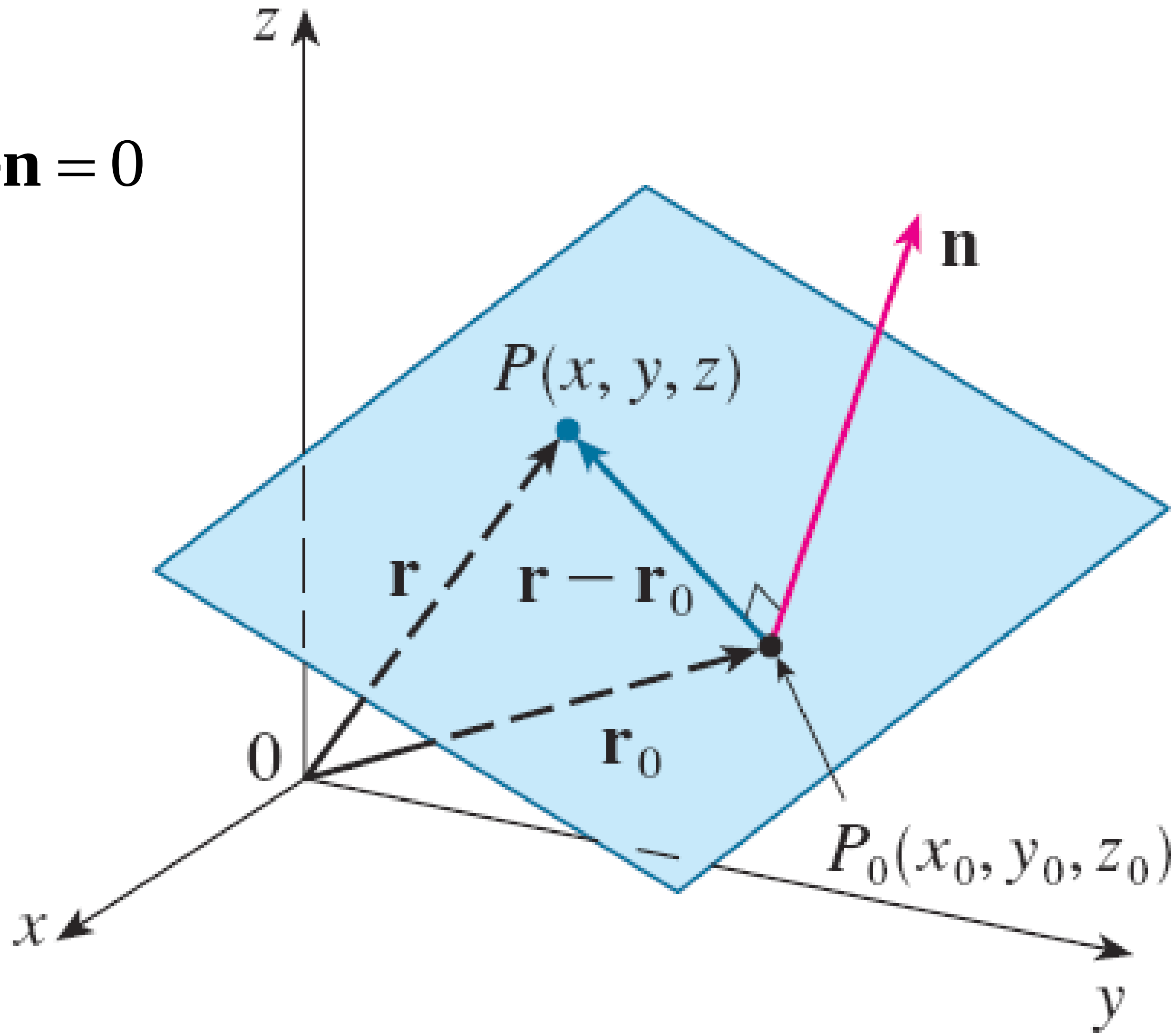

z
P0(x0, y0, z0)
a
P(x, y, z)
L
r0
r
v
O
x
y

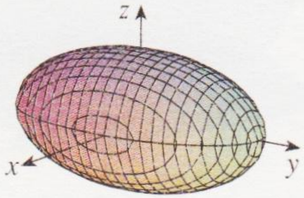
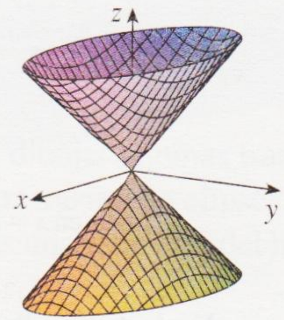
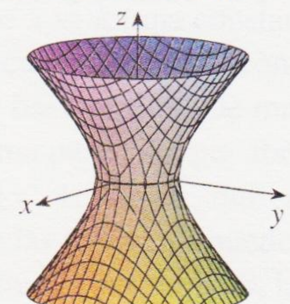
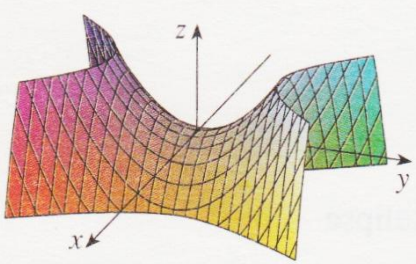
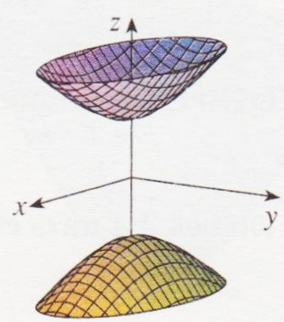
Plano: un punto y normal dados

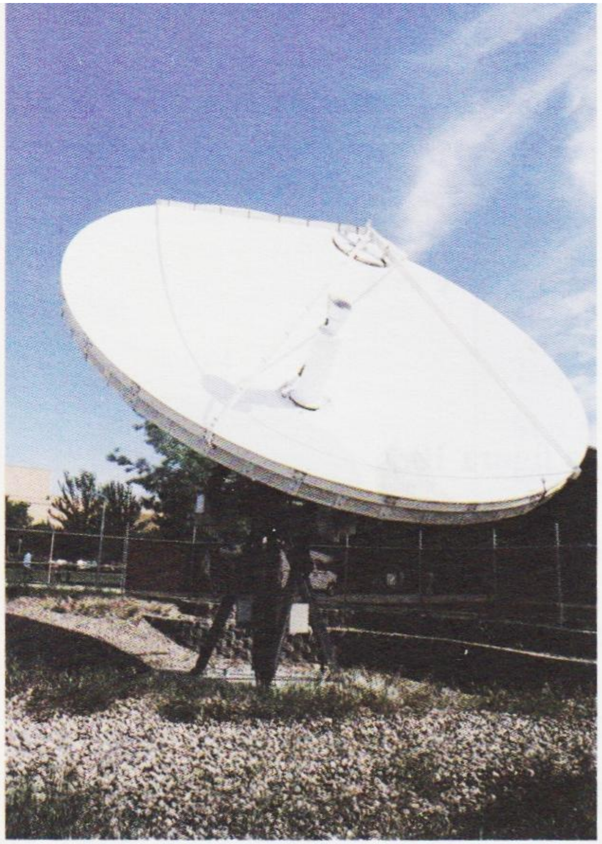


Plano: un punto y normal dados

$$\left(\mathbf{r}-\mathbf{r}_0\right)\bullet\mathbf{n}=0$$



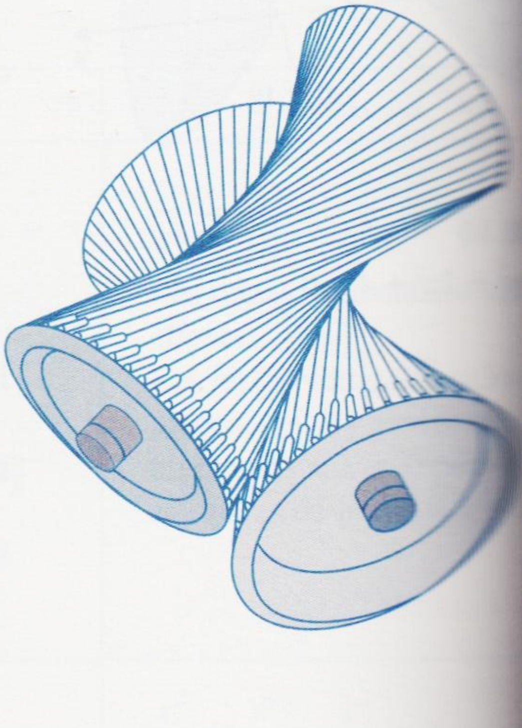
Superficie	Ecuación	Superficie	Ecuación
Elipsoide 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Todas las trazas son elipses. Si $a = b = c$, la elipsoide es una esfera.	Cono 	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Las trazas horizontales son elipses. Las trazas verticales en los planos $x = k$ y $y = k$ son hipérbolas si $k \neq 0$ pero son pares de líneas si $k = 0$.
Paraboloide elíptico 	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Las trazas horizontales son elipses. Las trazas verticales son parábolas. La variable elevada a la primera potencia indica el eje del paraboloide.	Hiperboloide de una hoja. 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Las trazas verticales son elipses. Las trazas verticales son hipérbolas. El eje de simetría corresponde a la variable cuyo coeficiente es negativo.
Paraboloide hiperbólico. 	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ Las trazas horizontales son hipérbolas. Las trazas verticales son parábolas. Se ilustra el caso donde $c < 0$.	Hiperboloide de dos hojas. 	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Las trazas horizontales en $z = k$ son elipses si $k > c$ o $k < -c$. Las trazas verticales son hipérbolas. Los dos signos menos indican dos hojas.



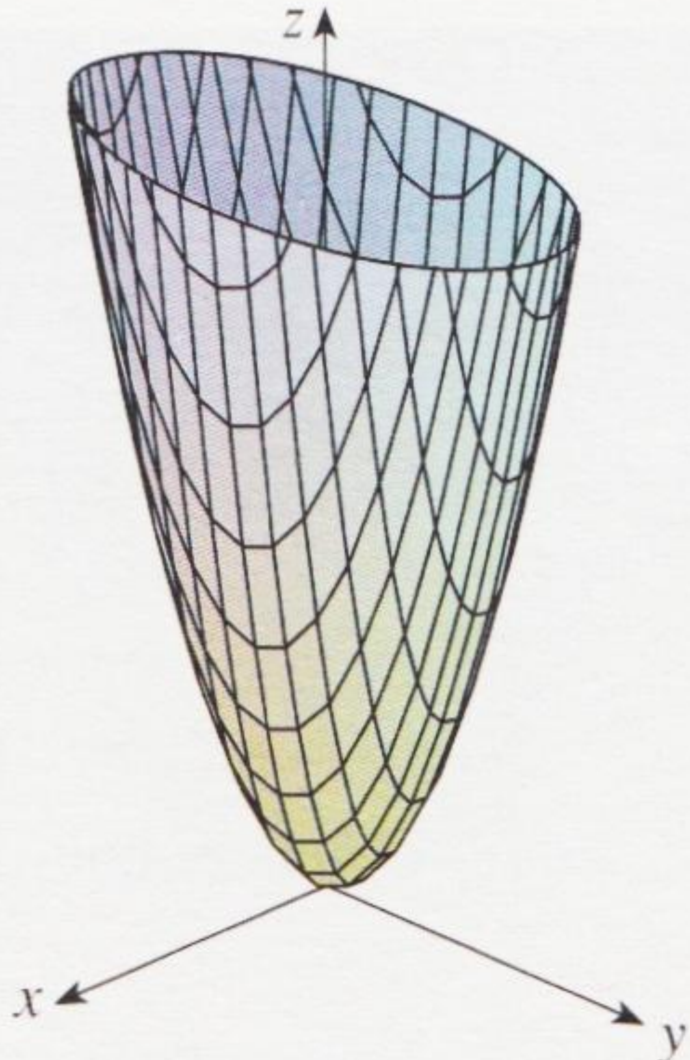
Una antena de disco refleja señales al foco de un paraboloide.

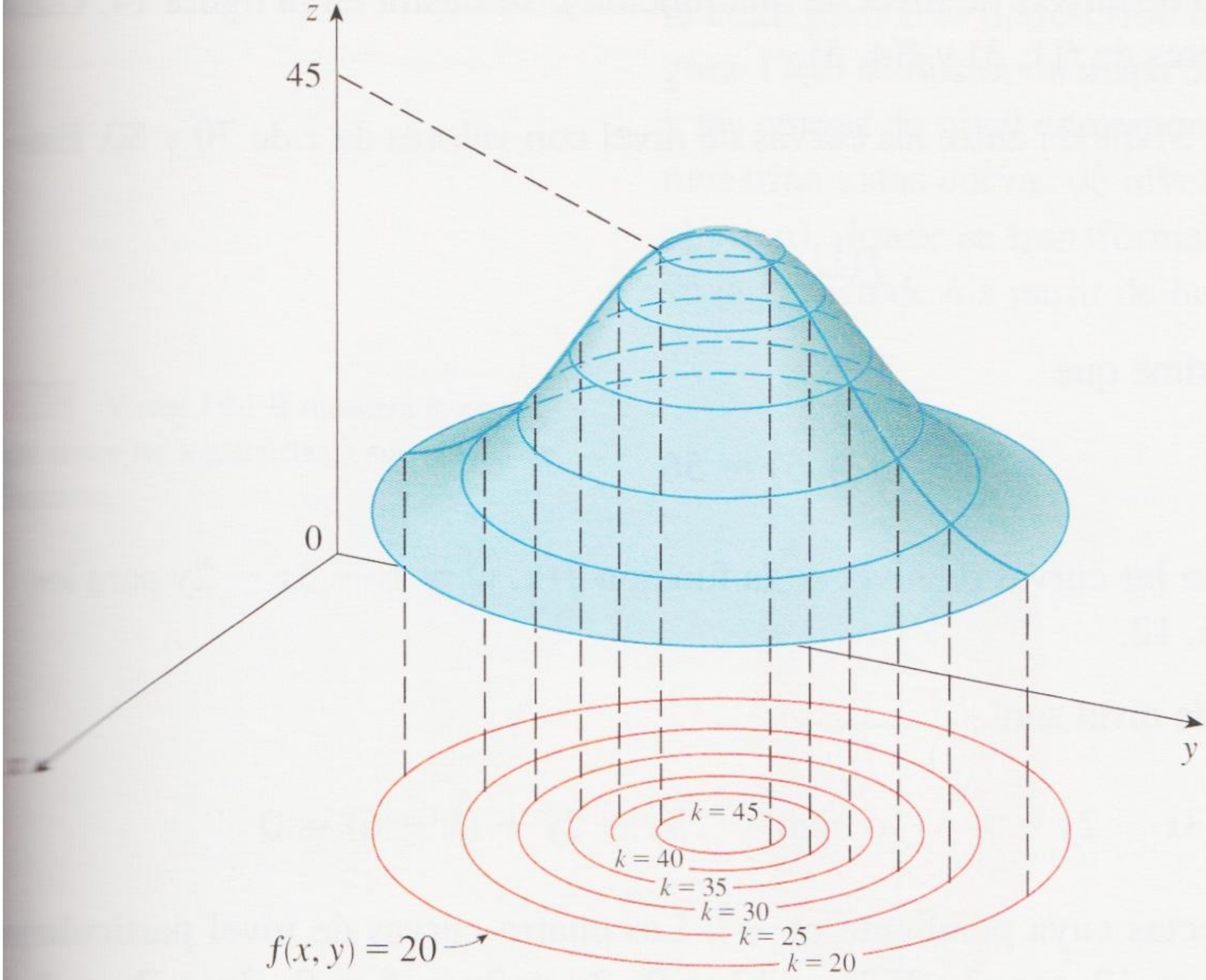
David Burnett / Photo Researchers, Inc

Los reactores nucleares tienen torres de enfriamiento en forma de hiperboloides.



Los hiperboloides producen transmisión por engranajes.


z
x
y


z
45
0
y
k = 45
k = 40
k = 35
k = 30
k = 25
k = 20
f(x, y) = 20

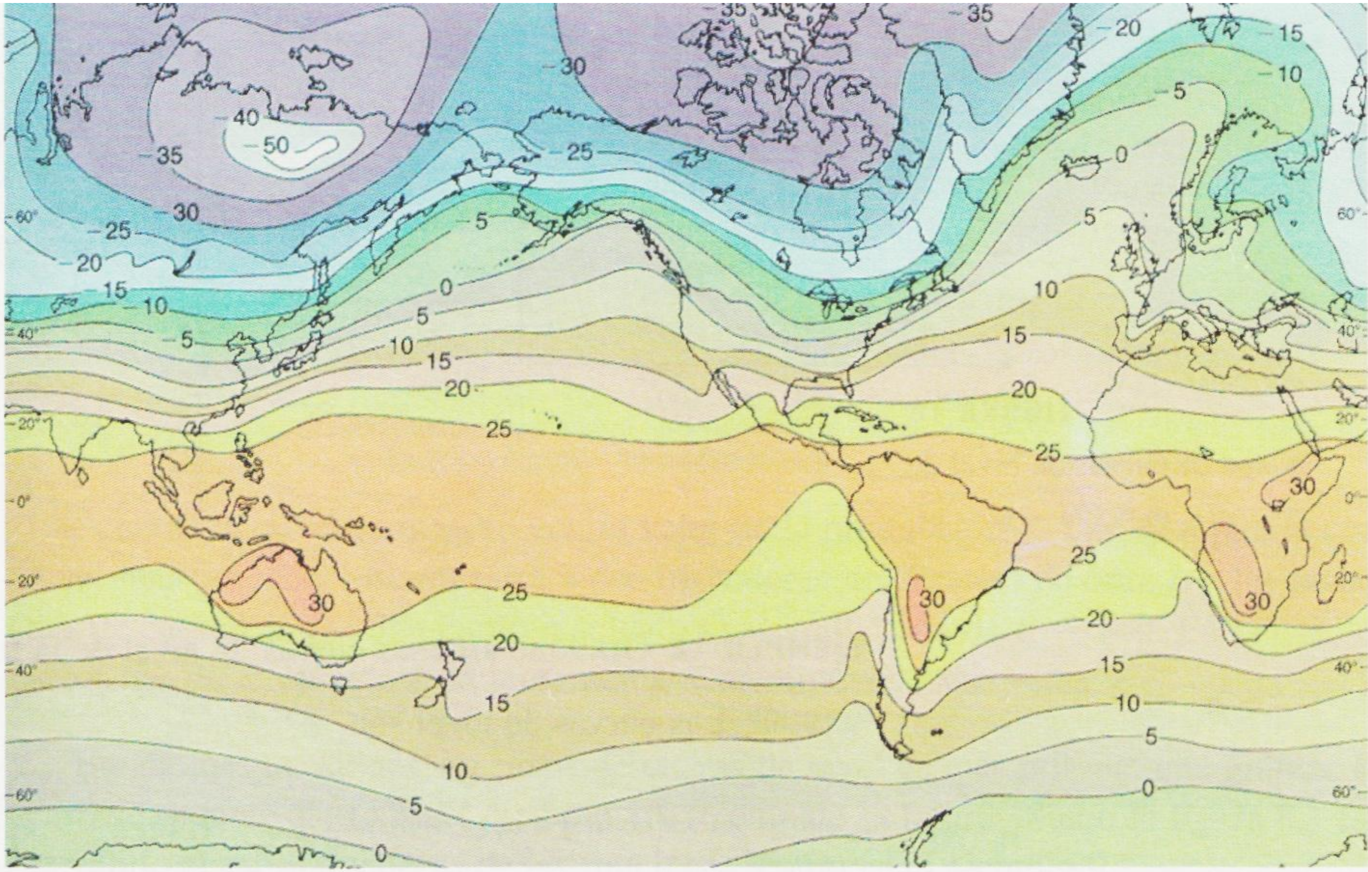
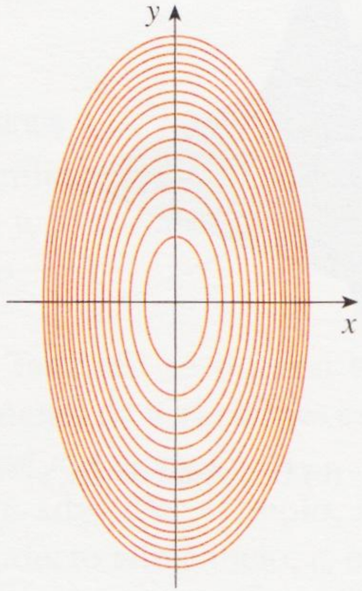


FIGURA 13

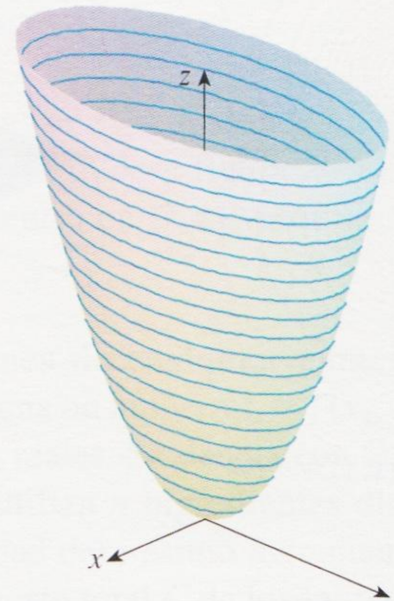
Temperaturas en el mundo a nivel medio del mar en el mes de enero dadas en grados Celsius

Tarbuck, *Atmosphere: Introduction to Meteorology*, 4th Edition, © 1989. Reimpreso con permiso de Pearson Education, Inc., Upper Saddle River, NJ.

Visual 14.1 B muestra la conec- ... entre las superficies y sus mapas de ...torno

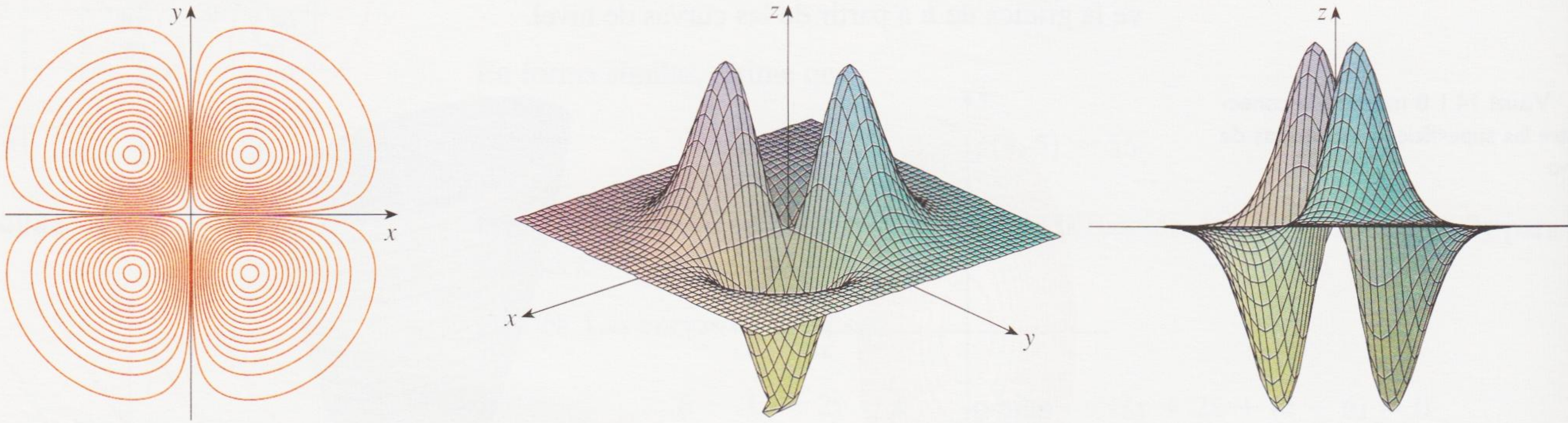


(a) Mapa de curvas de nivel



(b) Trazas horizontales, son curvas de nivel elevadas

FIGURA 17
La gráfica de $h(x, y) = 4x^2 + y^2$ se forma elevando las curvas de nivel



(a) Curvas de nivel de $f(x, y) = -xye^{-x^2-y^2}$

(b) Dos vistas de $f(x, y) = -xye^{-x^2-y^2}$

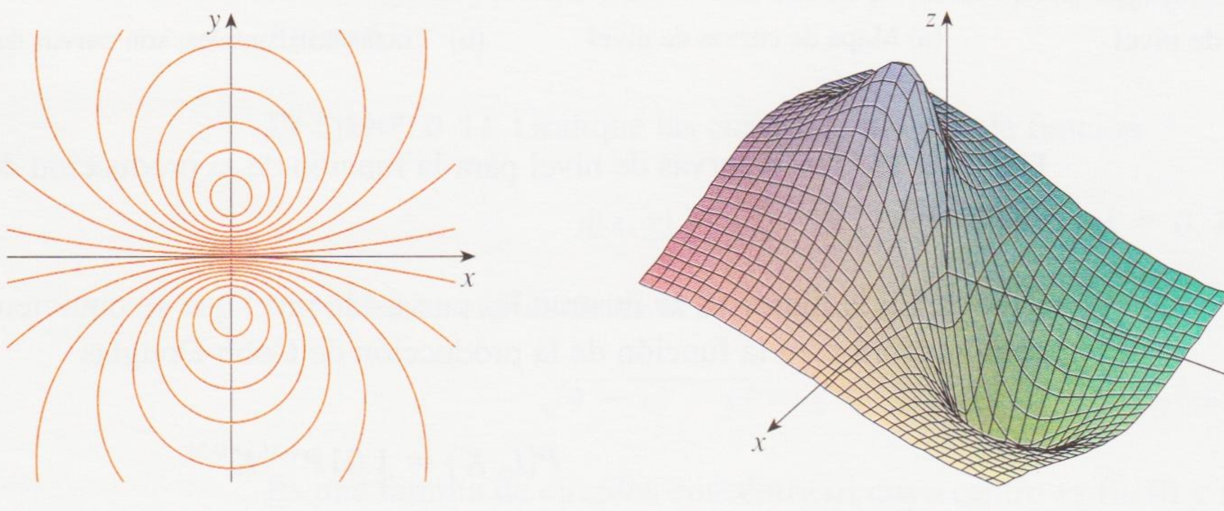
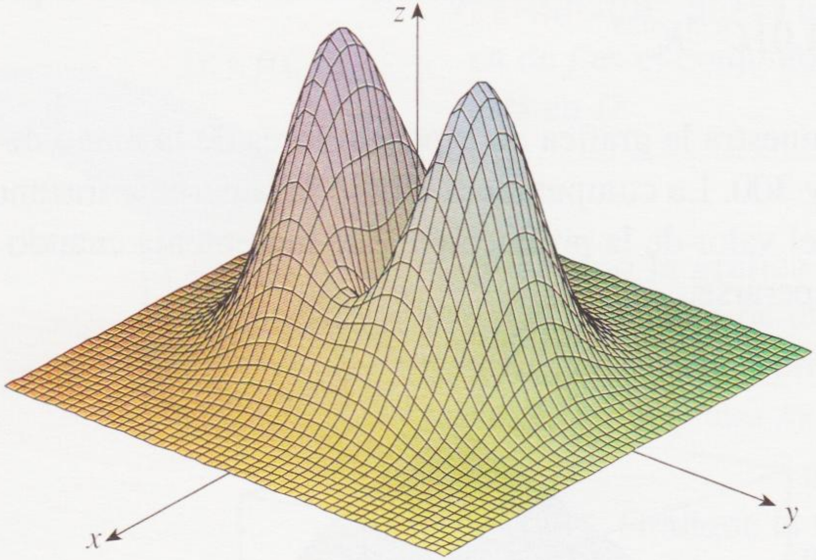


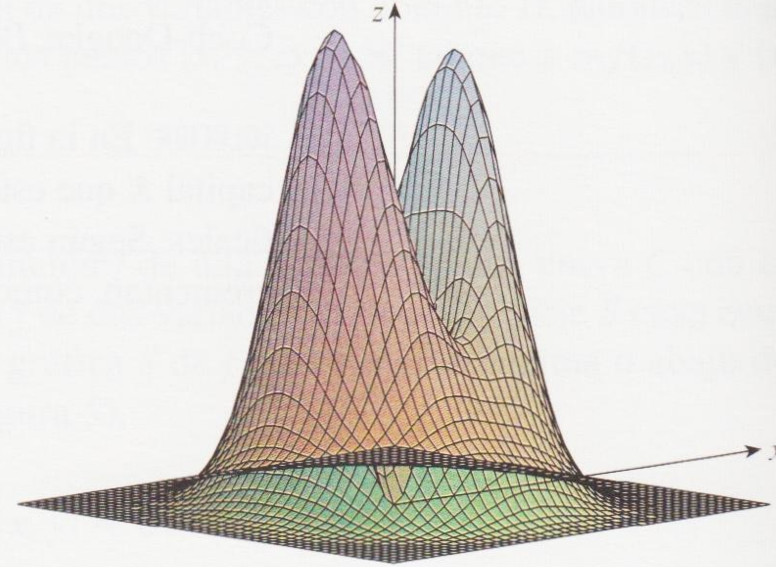
FIGURA 19

(c) Curvas de nivel de $f(x, y) = \dfrac{-3y}{x^2 + y^2 + 1}$

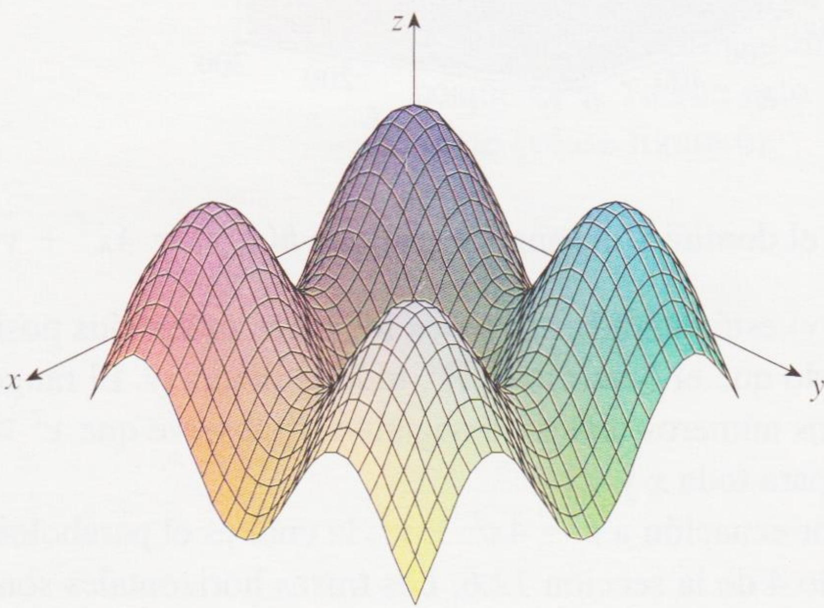
(d) $f(x, y) = \dfrac{-3y}{x^2 + y^2 + 1}$



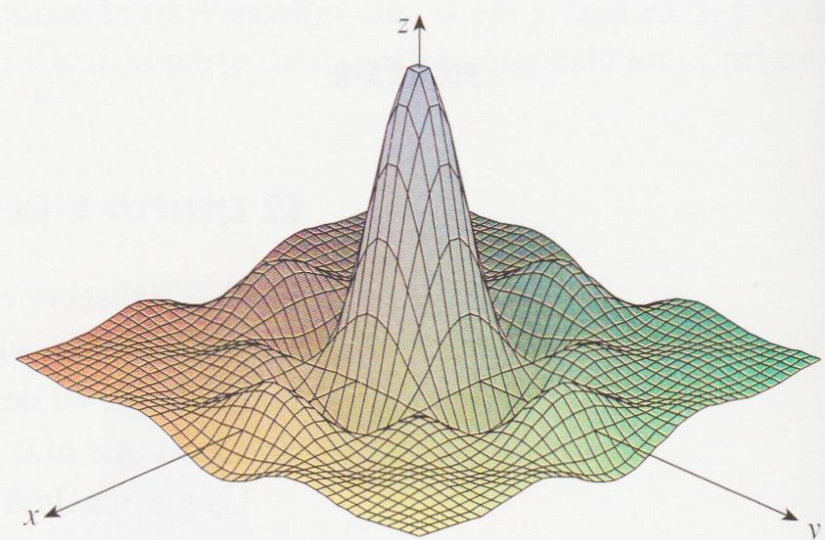
(a) $f(x, y) = (x^2 + 3y^2)e^{-x^2-y^2}$



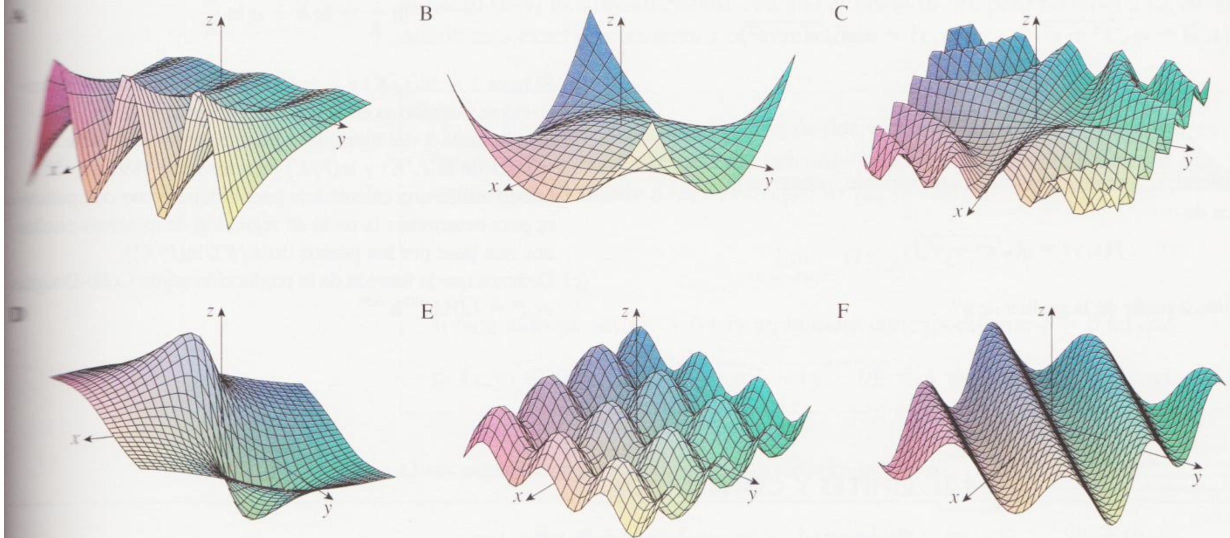
(b) $f(x, y) = (x^2 + 3y^2)e^{-x^2-y^2}$

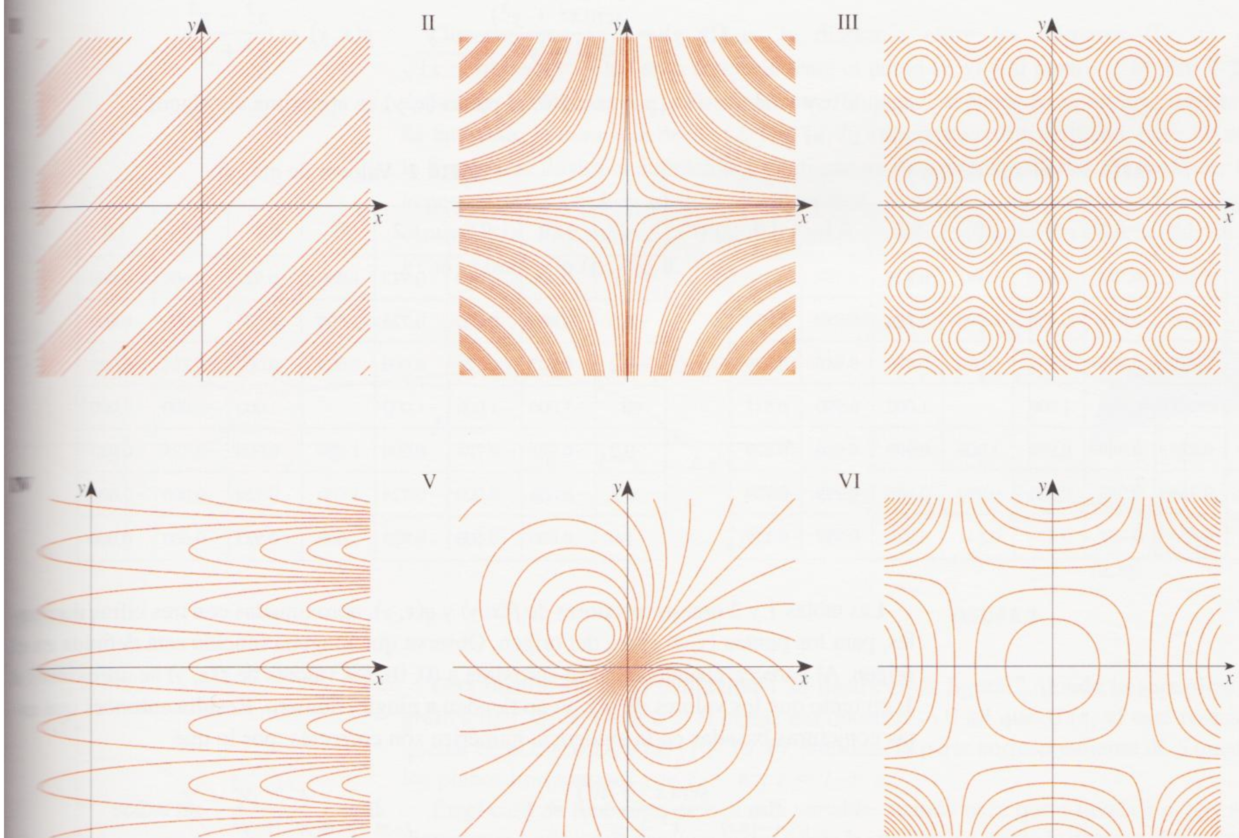


(c) $f(x, y) = \operatorname{sen} x + \operatorname{sen} y$



(d) $f(x, y) = \dfrac{\operatorname{sen} x \operatorname{sen} y}{xy}$


B
C
E
F
z
x
y

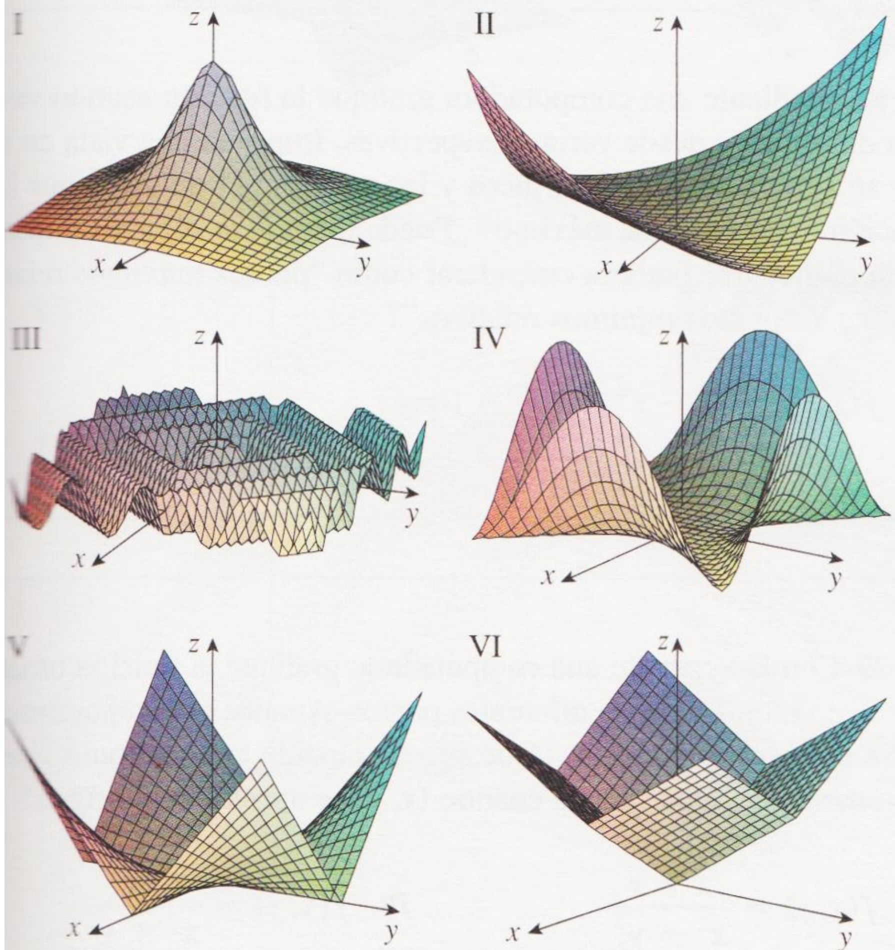

II
III
V
VI
x
y

Haga corresponder la función con su gráfica (marcadas de I a VI). Ofrezca razones por su elección.

(a) $f(x, y) = |x| + |y|$ (b) $f(x, y) = |xy|$

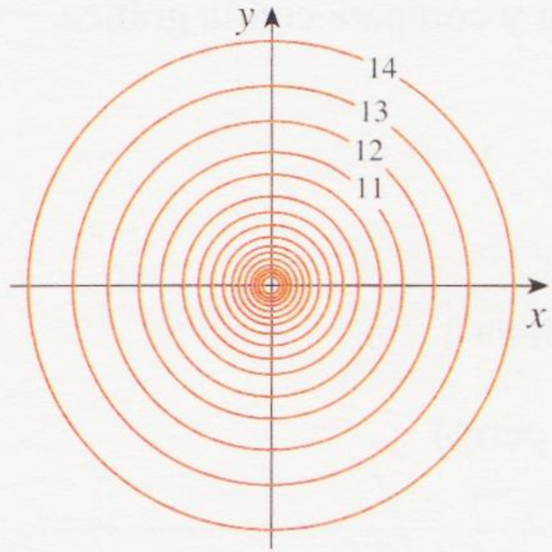
(c) $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$ (d) $f(x, y) = (x^2 - y^2)^2$

(e) $f(x, y) = (x - y)^2$ (f) $f(x, y) = \text{sen}(|x| + |y|)$

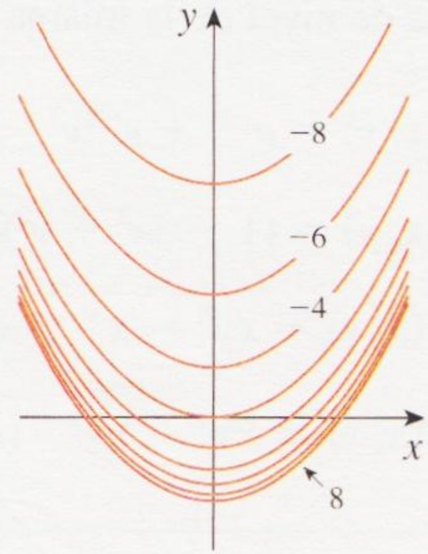


35–38 Se ilustra un mapa de curvas de nivel de una función. Apóyese en él para elaborar un esquema aproximado de la gráfica de f.

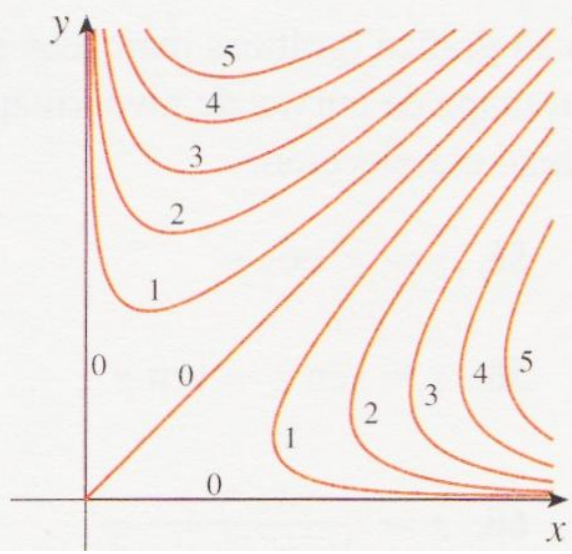
35.



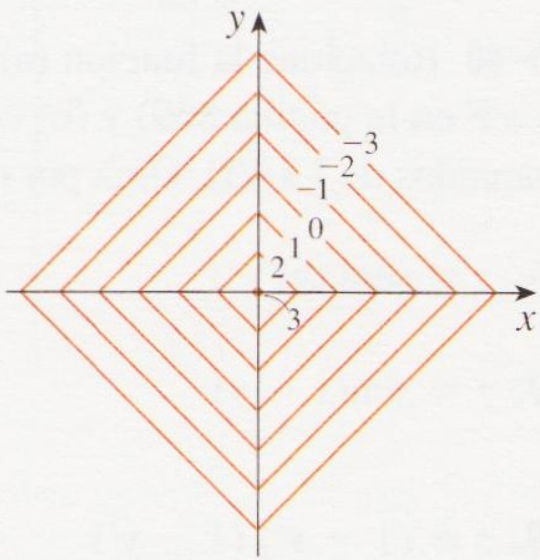
36.

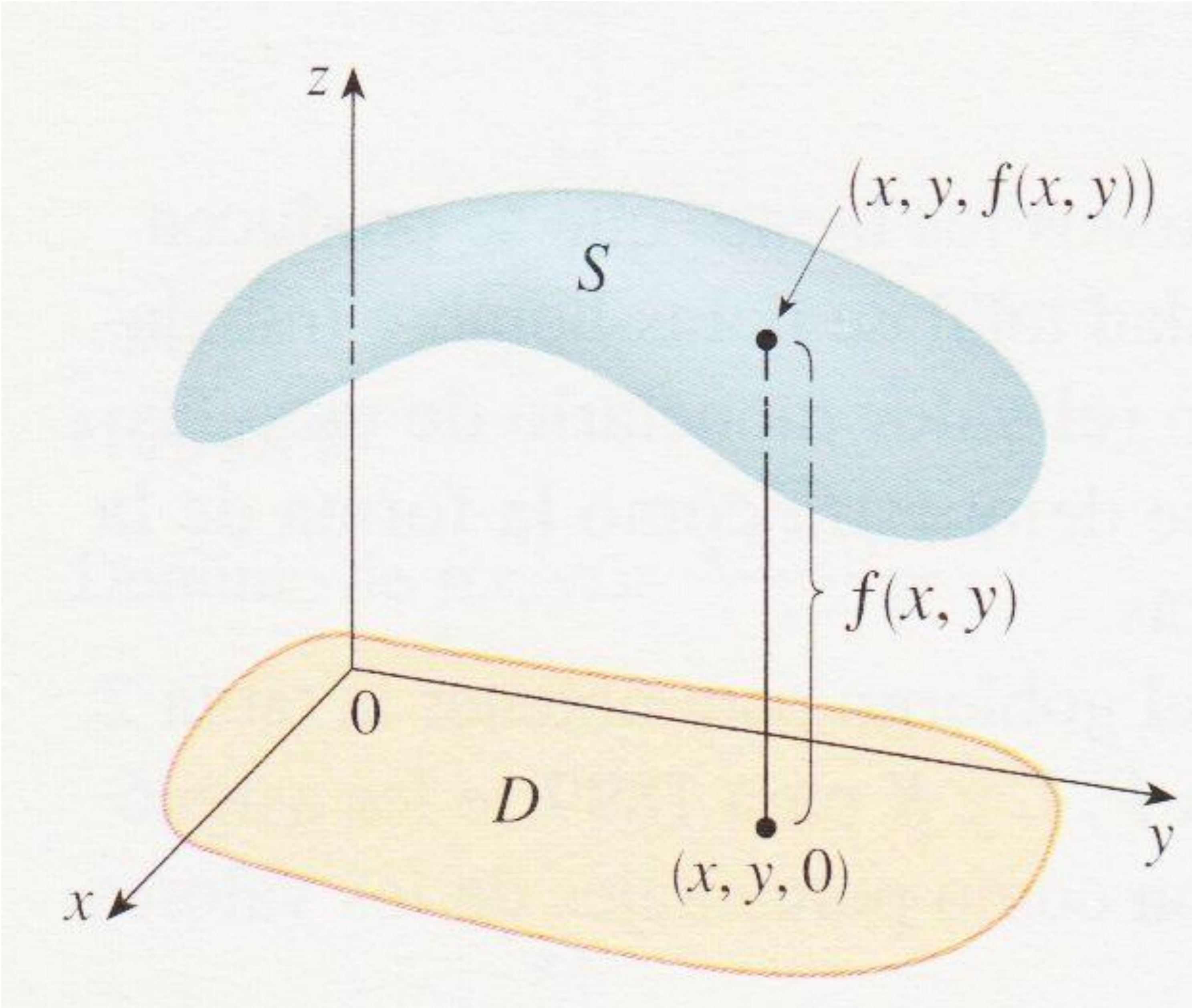


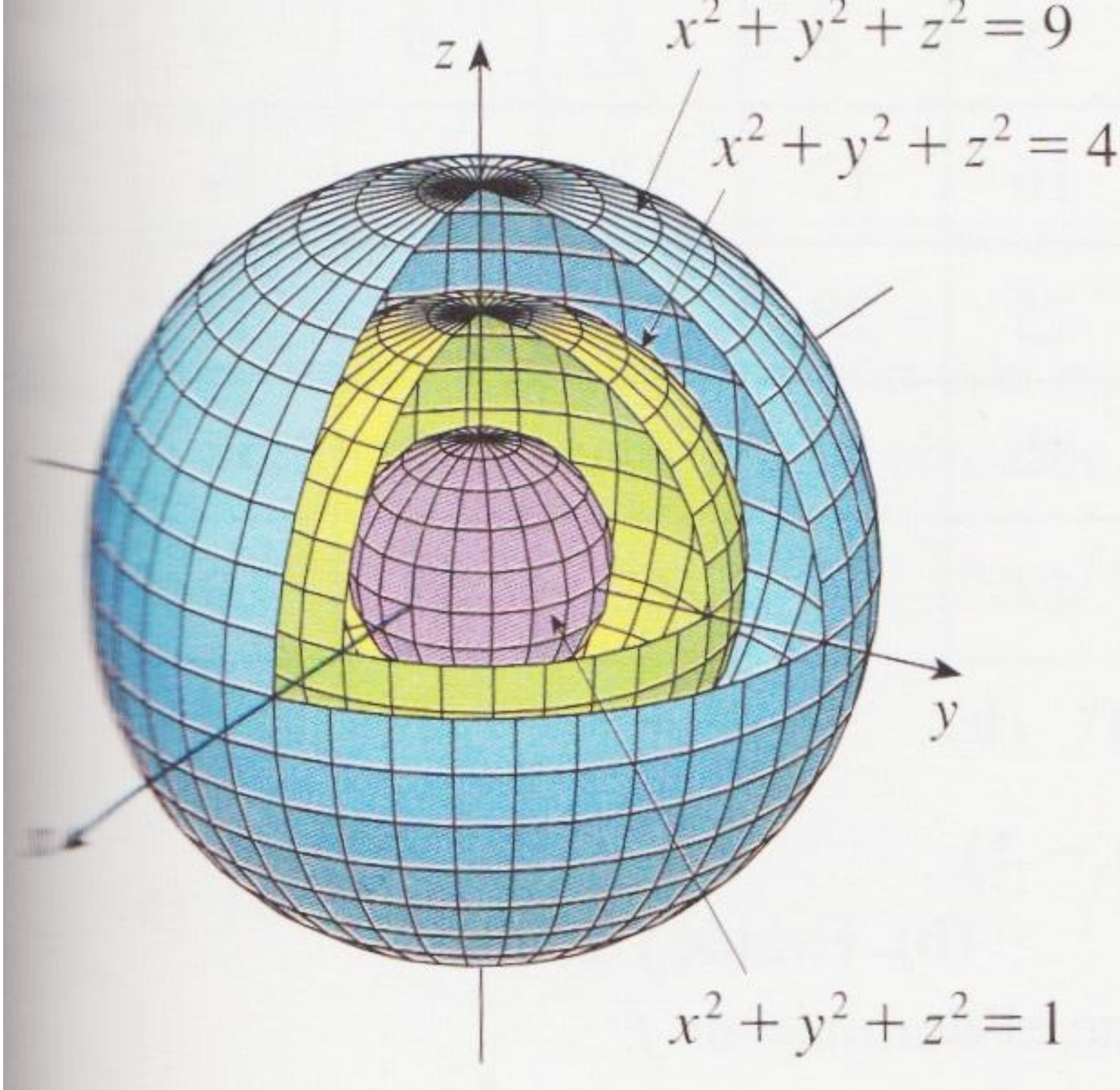
37.



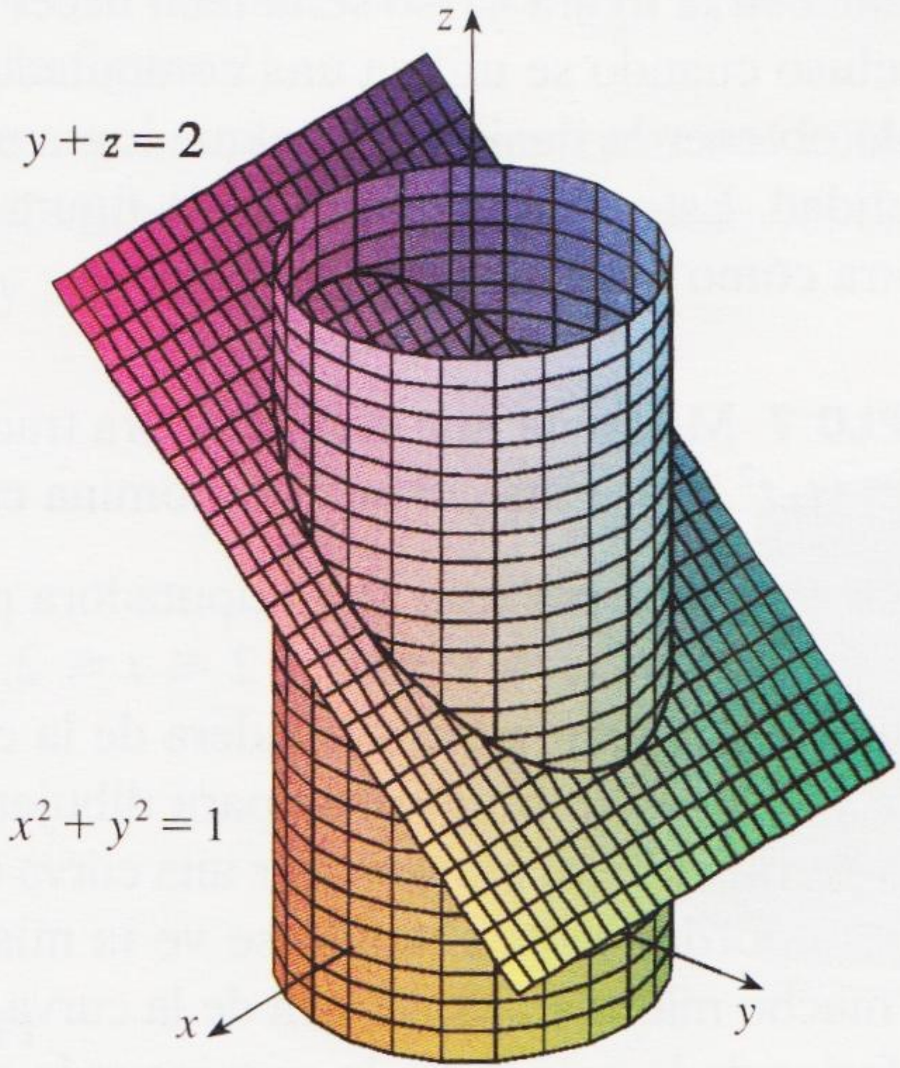
38.

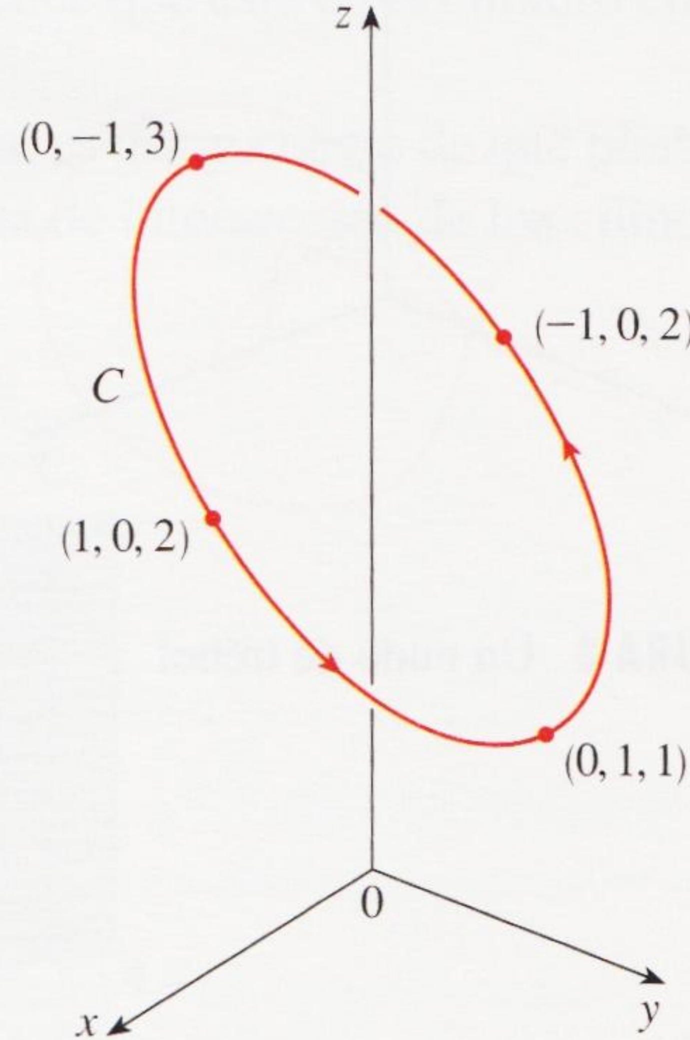



z
(x, y, f(x, y))
S
f(x, y)
0
D
y
(x, y, 0)
x



$x^2 + y^2 + z^2 = 9$
$x^2 + y^2 + z^2 = 4$
z
y
$x^2 + y^2 + z^2 = 1$


z
y + z = 2
x^2 + y^2 = 1
x
y


z
(0, −1, 3)
(−1, 0, 2)
C
(1, 0, 2)
(0, 1, 1)
0
x
y

Plano: definido por tres puntos

con coordenadas

$P(x_p, y_p, z_p)$

$Q(x_Q, y_Q, z_Q)$

$R(x_R, y_R, z_R)$

tome

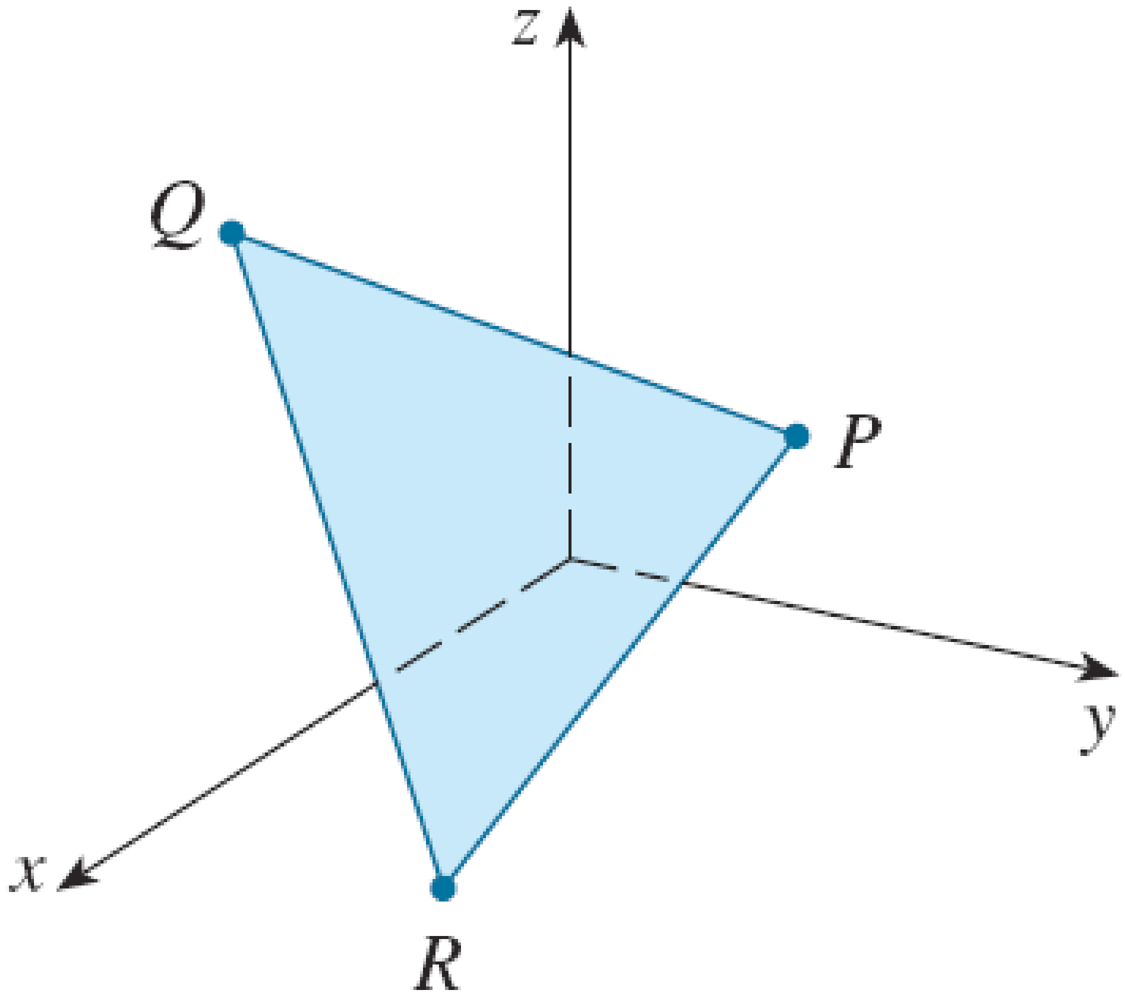
$\mathbf{r}_0 = P$

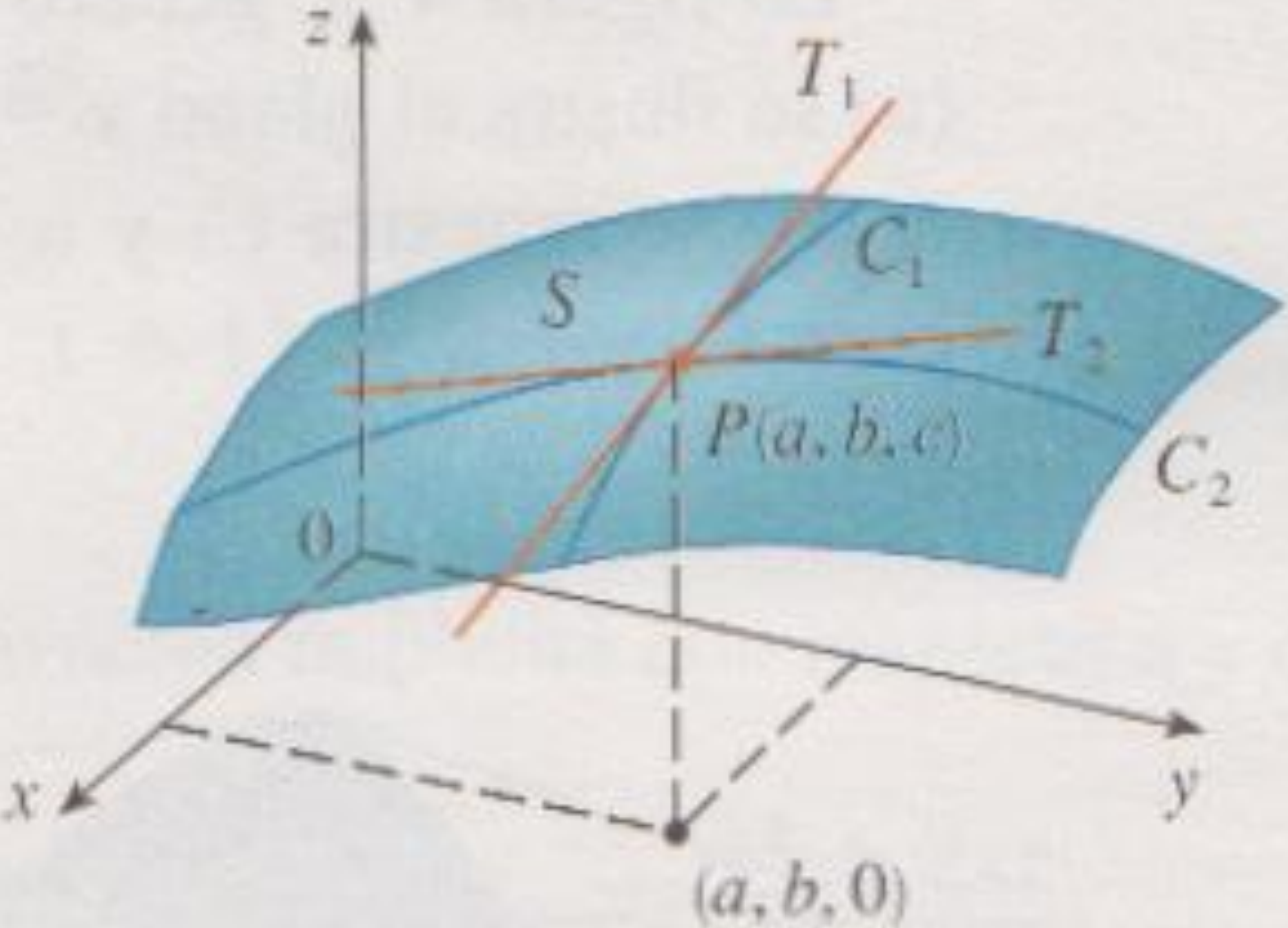
$\mathbf{r}_1 = PQ = Q - P$

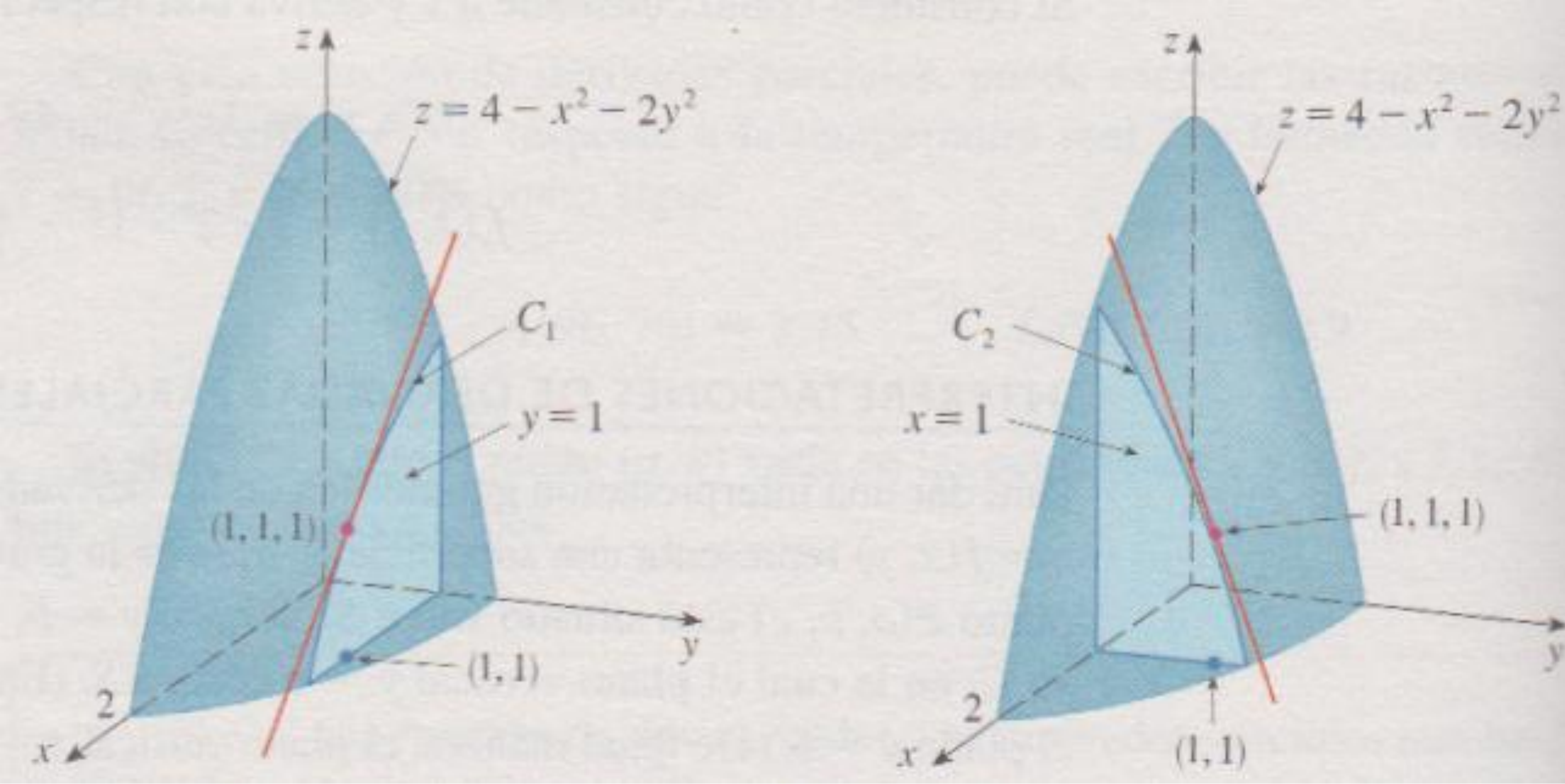
$\mathbf{r}_2 = PR = R - P$

por lo que

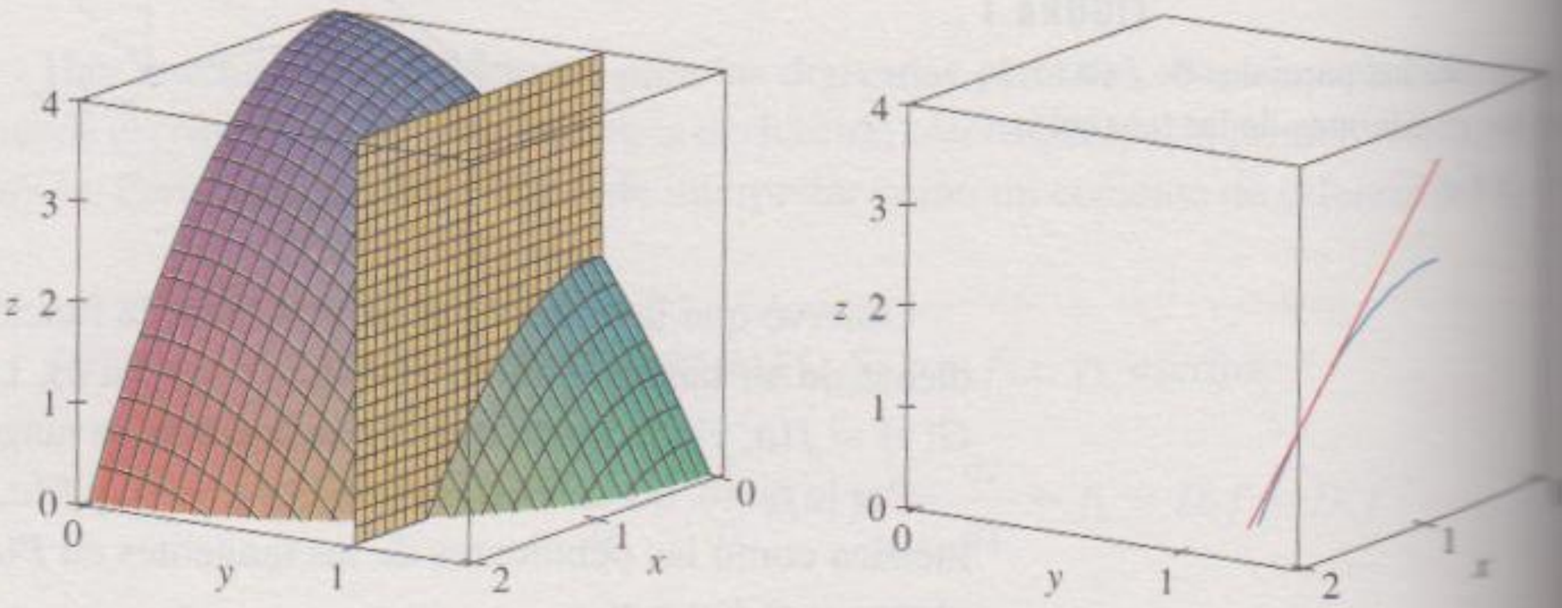
$\mathbf{n} = \mathbf{r}_1 \times \mathbf{r}_2$



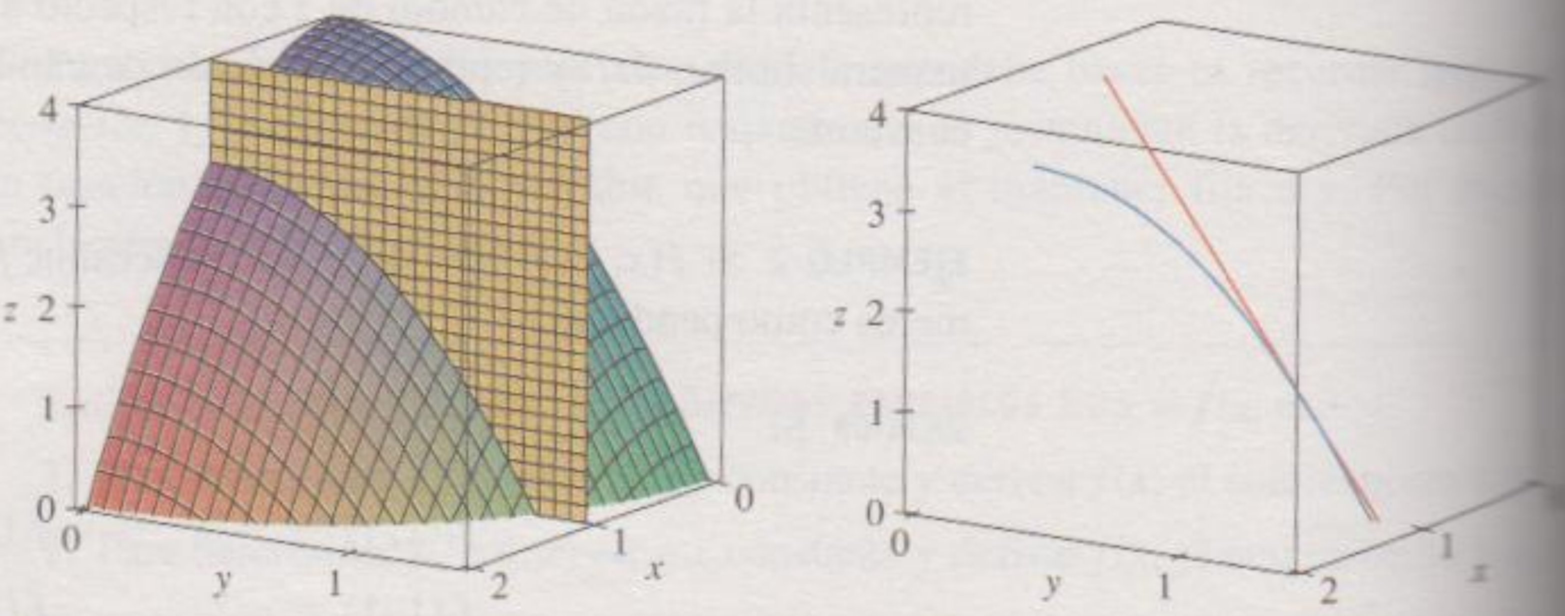

z
T_1
C_1
S
T_2
$P(a, b, c)$
C_2
0
x
y
$(a, b, 0)$

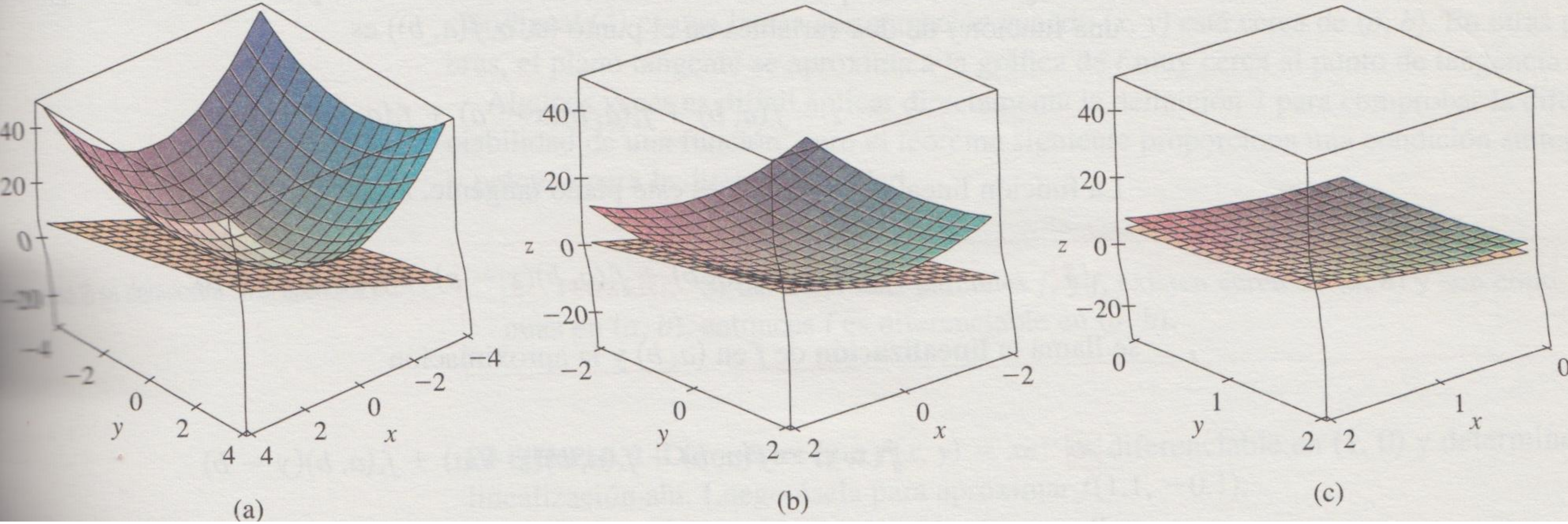


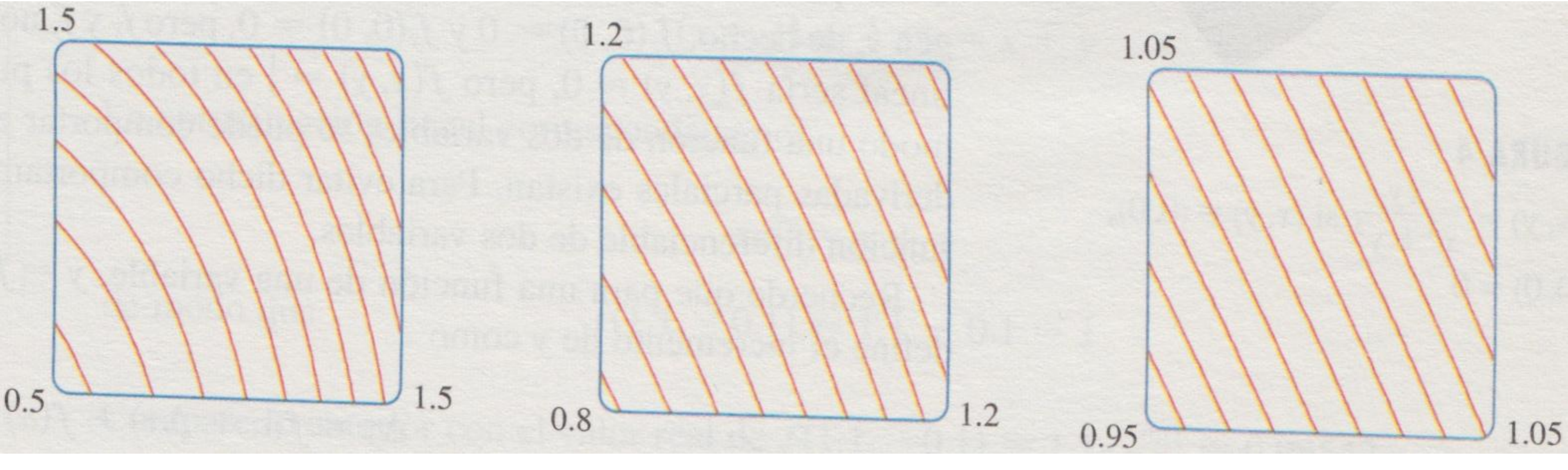
z
$z = 4 - x^2 - 2y^2$
C_1
$y = 1$
(1, 1, 1)
(1, 1)
2
x
y
z
$z = 4 - x^2 - 2y^2$
C_2
$x = 1$
(1, 1, 1)
(1, 1)
2
x
y

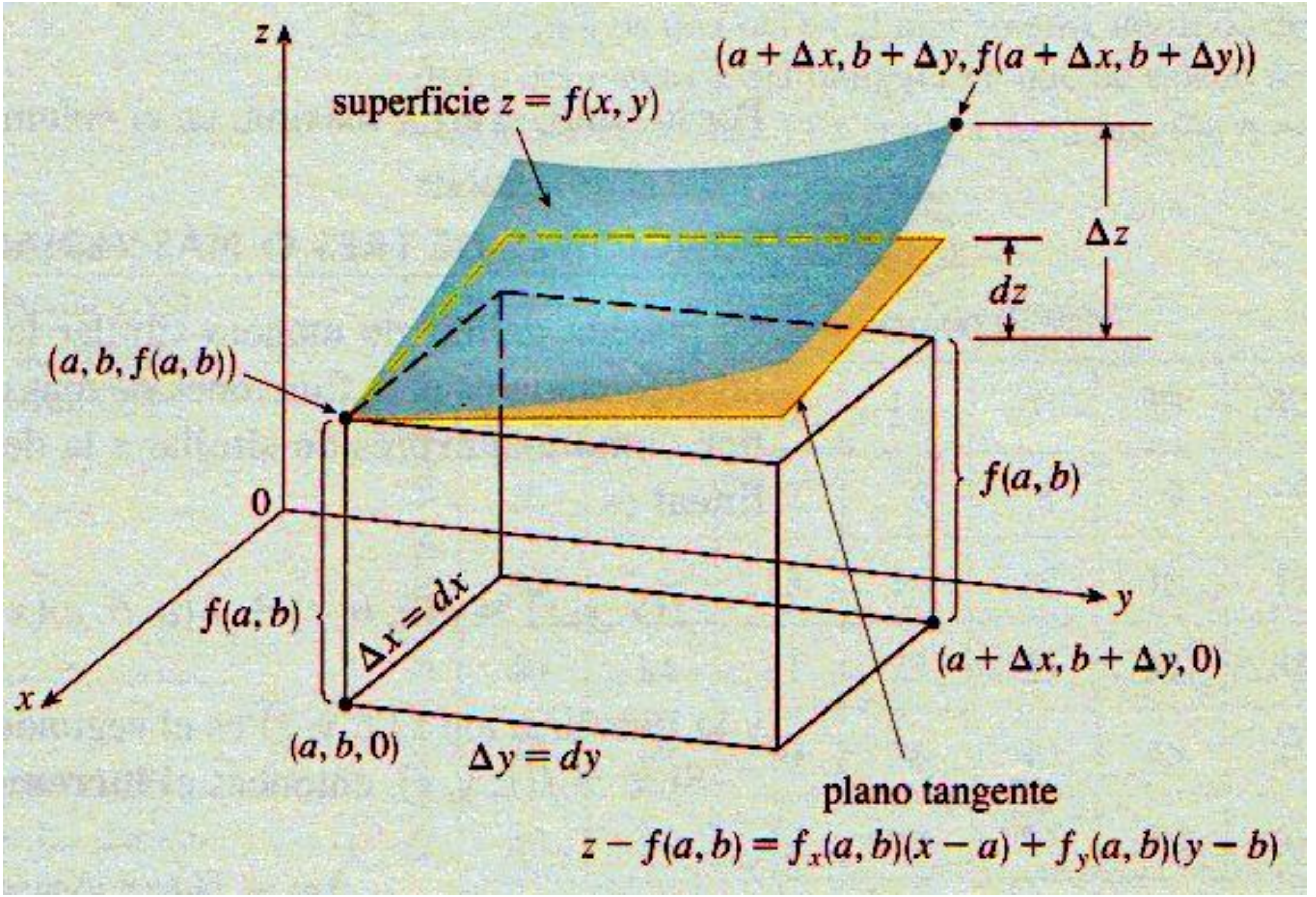


(a) (b)




40
20
0
−20
z
−4
−2
0
2
4
y
x
1
(a)
(b)
(c)


1.5
0.5
1.5
1.2
0.8
1.2
1.05
0.95
1.05



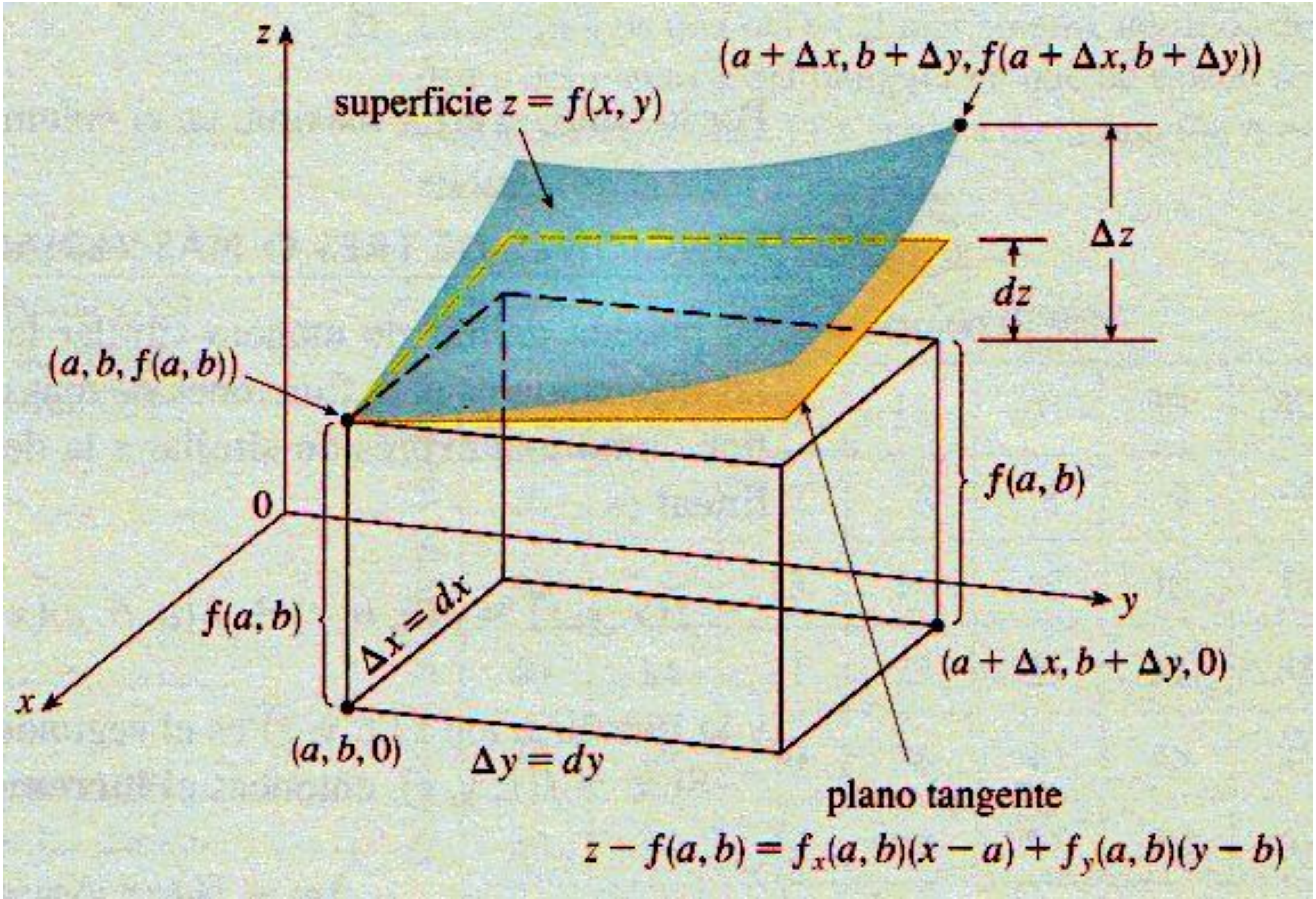
z
$(a + \Delta x, b + \Delta y, f(a + \Delta x, b + \Delta y))$
superficie $z = f(x, y)$
Δz
dz
$(a, b, f(a, b))$
$f(a, b)$
0
y
$f(a, b)$
$\Delta x = dx$
$(a + \Delta x, b + \Delta y, 0)$
x
$(a, b, 0)$
$\Delta y = dy$
plano tangente
$z - f(a, b) = f_x(a, b)(x - a) + f_y(a, b)(y - b)$

$2D$

$$dz = f_x(x,y)\,dx + f_y(x,y)\,dy = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$3D$

$$dw = f_x(x,y,z)\,dx + f_y(x,y,z)\,dy + f_z(x,y,z)\,dz =$$



$2D$

$$dz = f_x(x,y)\,dx + f_y(x,y)\,dy = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$3D$

$$dw = f_x(x,y,z)\,dx + f_y(x,y,z)\,dy + f_z(x,y,z)\,dz =$$

$$= \frac{\partial w(x,y,z)}{\partial x}dx + \frac{\partial w(x,y,z)}{\partial y}dy + \frac{\partial w(x,y,z)}{\partial z}dz$$

direccional

$2D$

$\mathbf{u} = (a,b) \leftarrow unitario$

$$D_{\mathbf{u}} f(x,y) = f_x(x,y)\,a + f_y(x,y)\,b$$

$\mathbf{u} = (\cos\alpha, \sin\alpha)$

$$D_{\mathbf{u}} f(x,y) = f_x(x,y)\cos\alpha + f_y(x,y)\sin\alpha$$

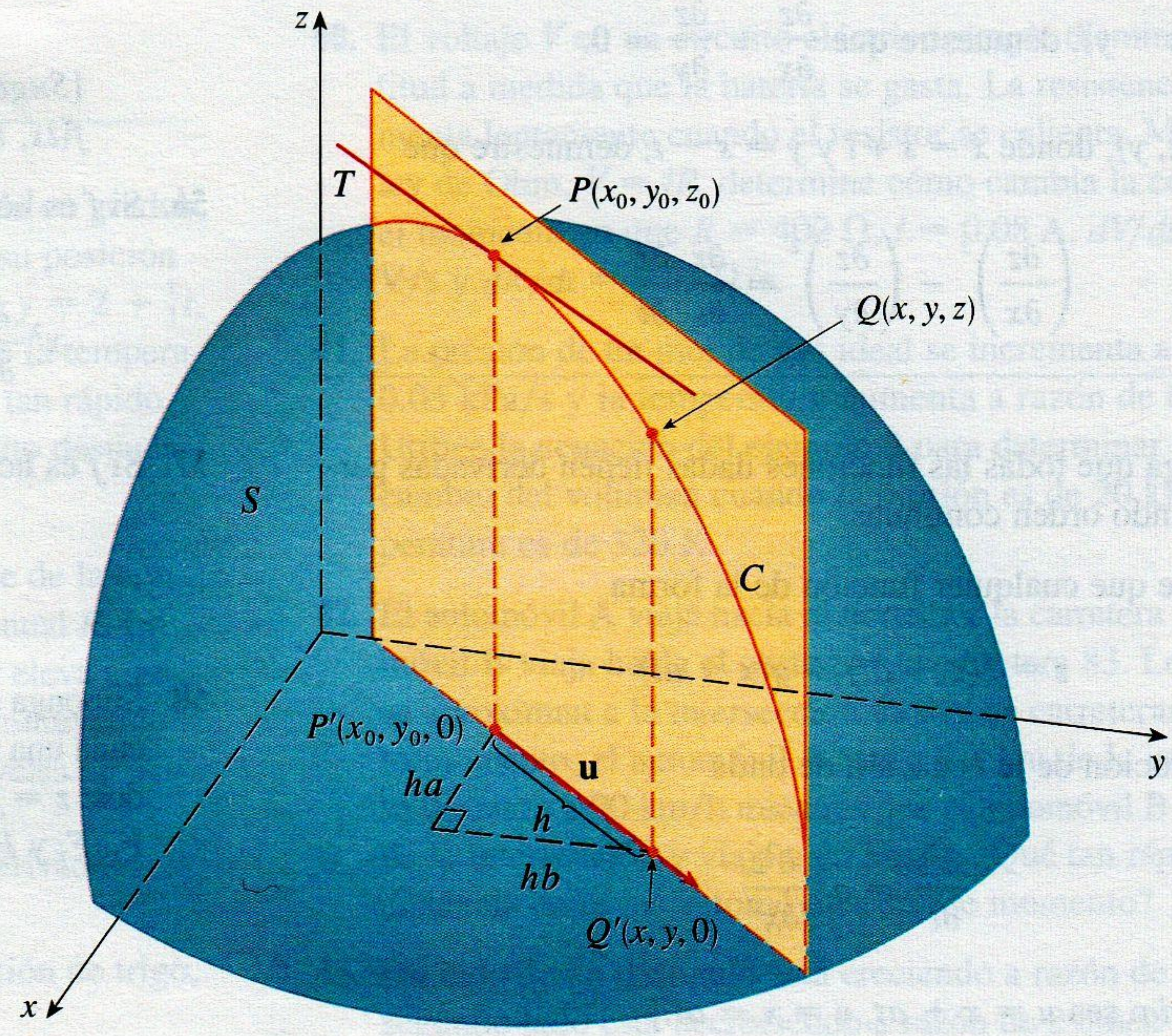
$$D_{\mathbf{u}} f(x,y) = f_x(x,y)\,a + f_y(x,y)\,b = \left(f_x, f_y\right)\bullet(a,b) = \left(f_x, f_y\right)\bullet\mathbf{u}$$

defina

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j}$$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$$

$$\therefore D_{\mathbf{u}} f(x,y) = \nabla f \bullet \mathbf{u}$$



z
T
P(x0, y0, z0)
Q(x, y, z)
S
C
P′(x0, y0, 0)
u
ha
h
hb
Q′(x, y, 0)
y
x

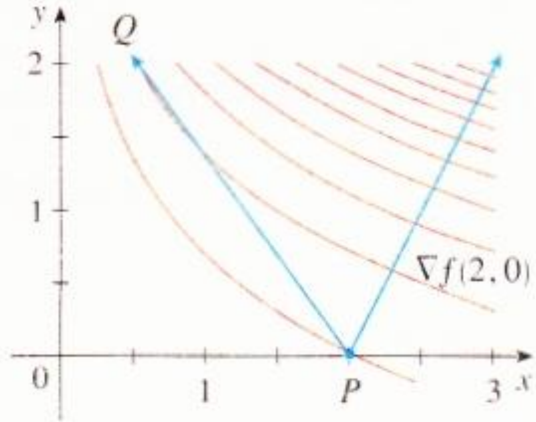
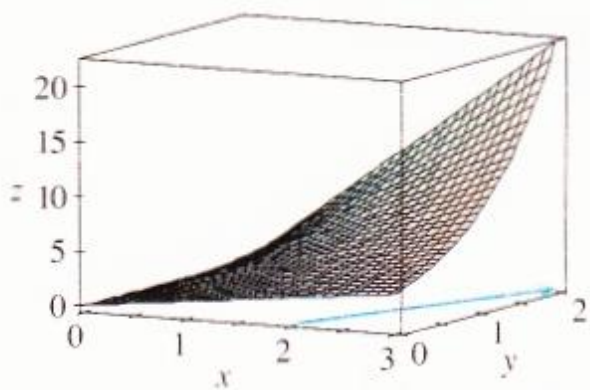
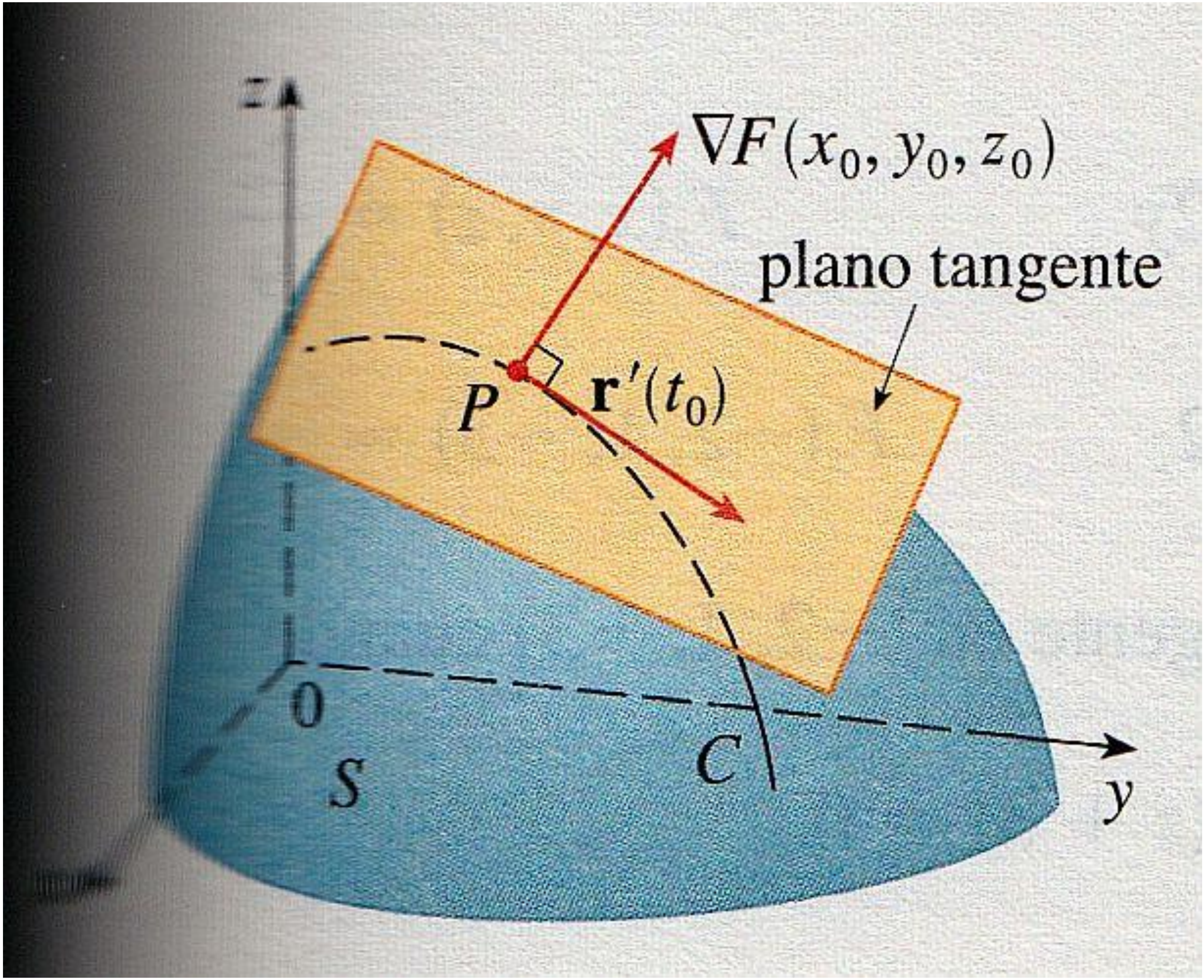


FIGURA 7

En $(2, 0)$ la función del ejemplo 6 se incrementa más rápido en la dirección del vector gradiente $\nabla f(2, 0) = \langle 1, 2 \rangle$. Observe que según la figura 7 este vector, al parecer, es perpendicular a la curva de nivel que pasa por $(2, 0)$. En la figura 8 se ilustra la gráfica de f y el vector gradiente.




z
$\nabla F(x_0, y_0, z_0)$
plano tangente
P
$\mathbf{r}'(t_0)$
0
S
C
y

$F(x, y, z) = 0 \leftarrow$ superficie S

$f(x, y) = k \leftarrow$ curva de nivel C que pasa por $P(x_0, y_0)$ y por tanto esta en S

$\mathbf{r}(t) = \langle x(t), y(t) \rangle \leftarrow$ curva C descrita vectorialmente

Sea t_0 el valor del parametro que corresponde a P

$\therefore \ \mathbf{r}(t_0) = \langle x_0, y_0 \rangle$ & $f\left(x(t), y(t)\right) = k$

y sean x, y funciones diferenciables de t y f diferenciable

$$\therefore \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} = 0$$

pero

$\nabla f = (f_x, f_y)$ y $\mathbf{r}'(t) = \langle x'(t), y'(t) \rangle$

$$\Rightarrow \boxed{\nabla f \bullet \mathbf{r}'(t) = 0}$$

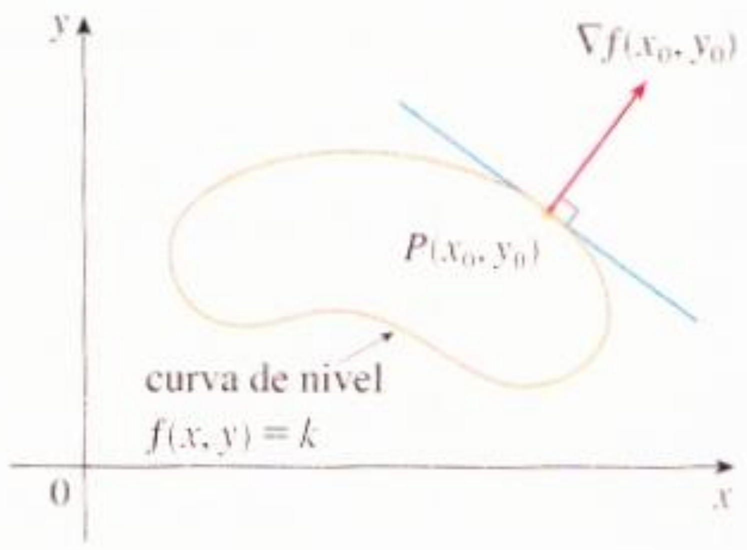
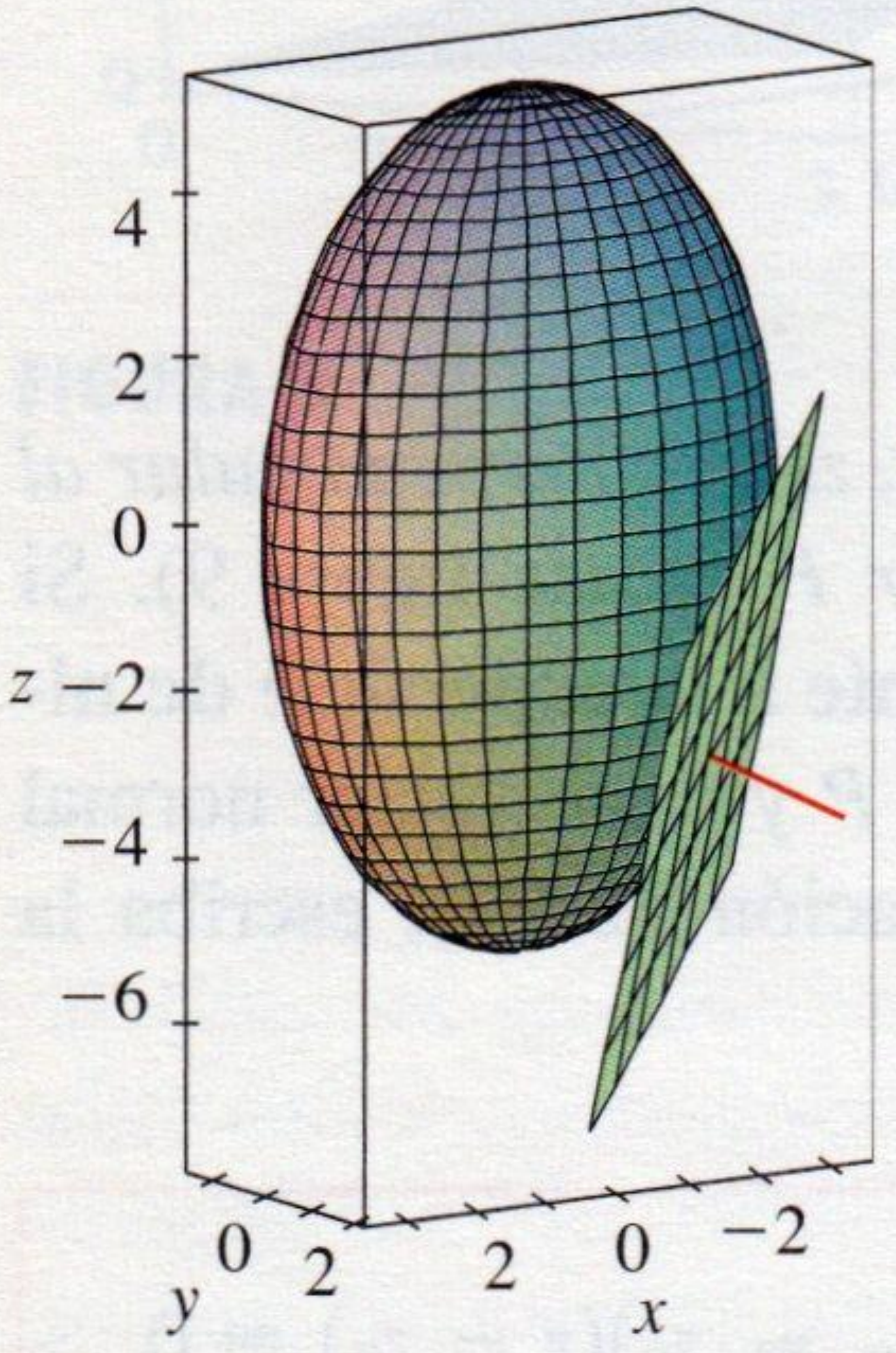


FIGURA 11

en particular

$\nabla f(x_0, y_0) \bullet \mathbf{r}'(t_0) = 0$

el vector gradiente es perperndicular al vector tangente en cualquier curva C en S que pasa por P



4
2
0
z −2
−4
−6
0
2
y
2
0
−2
x

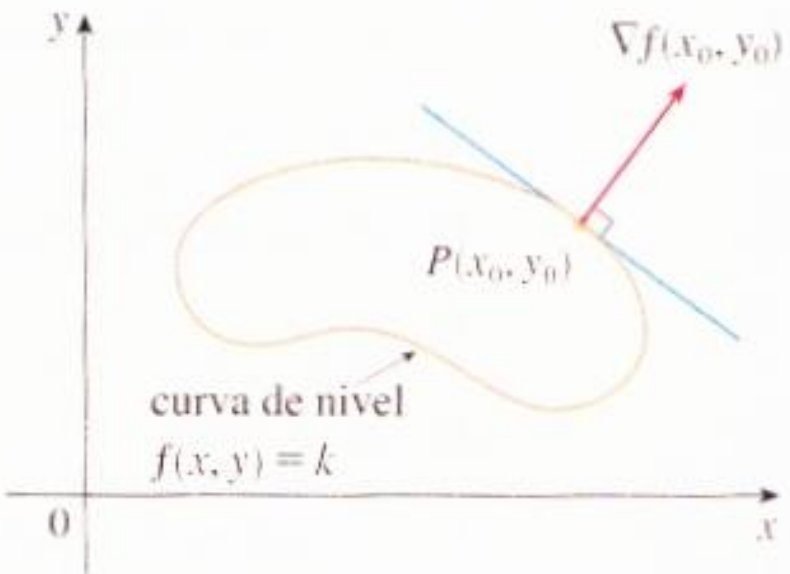


FIGURA 11

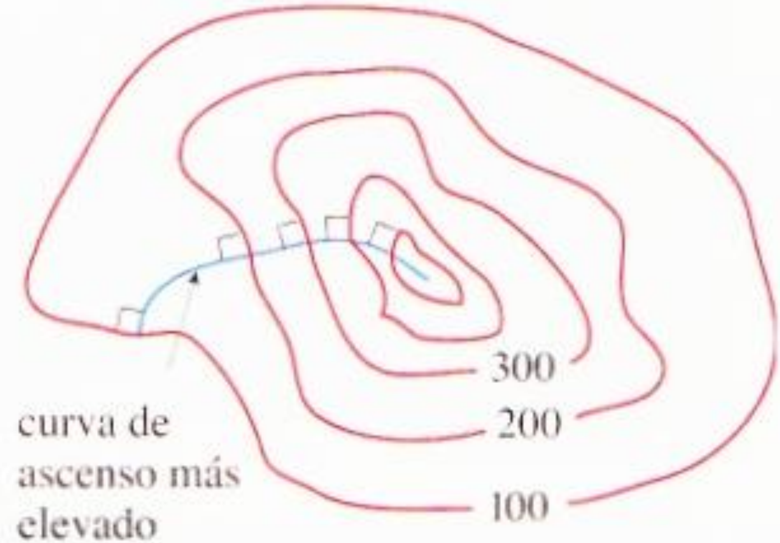
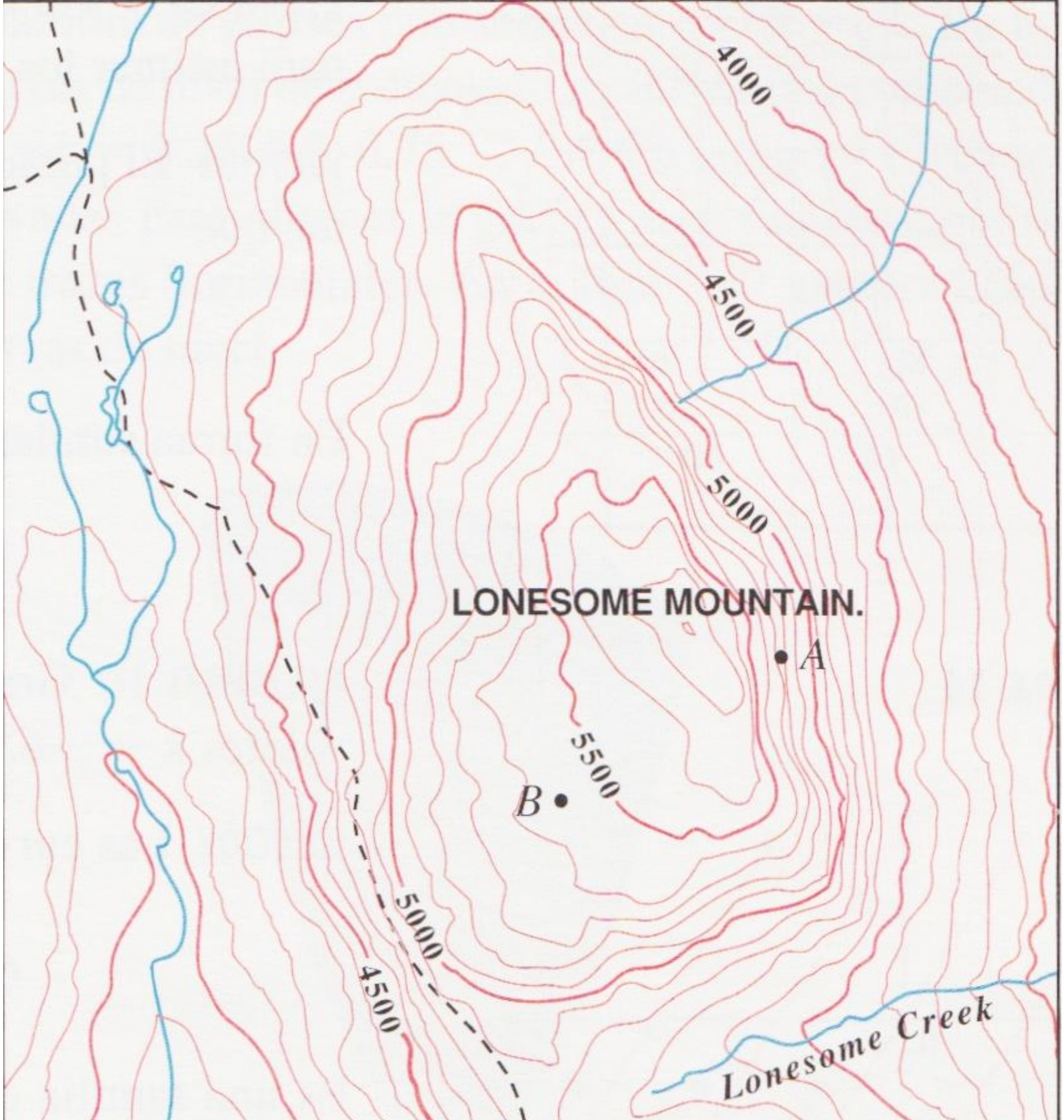
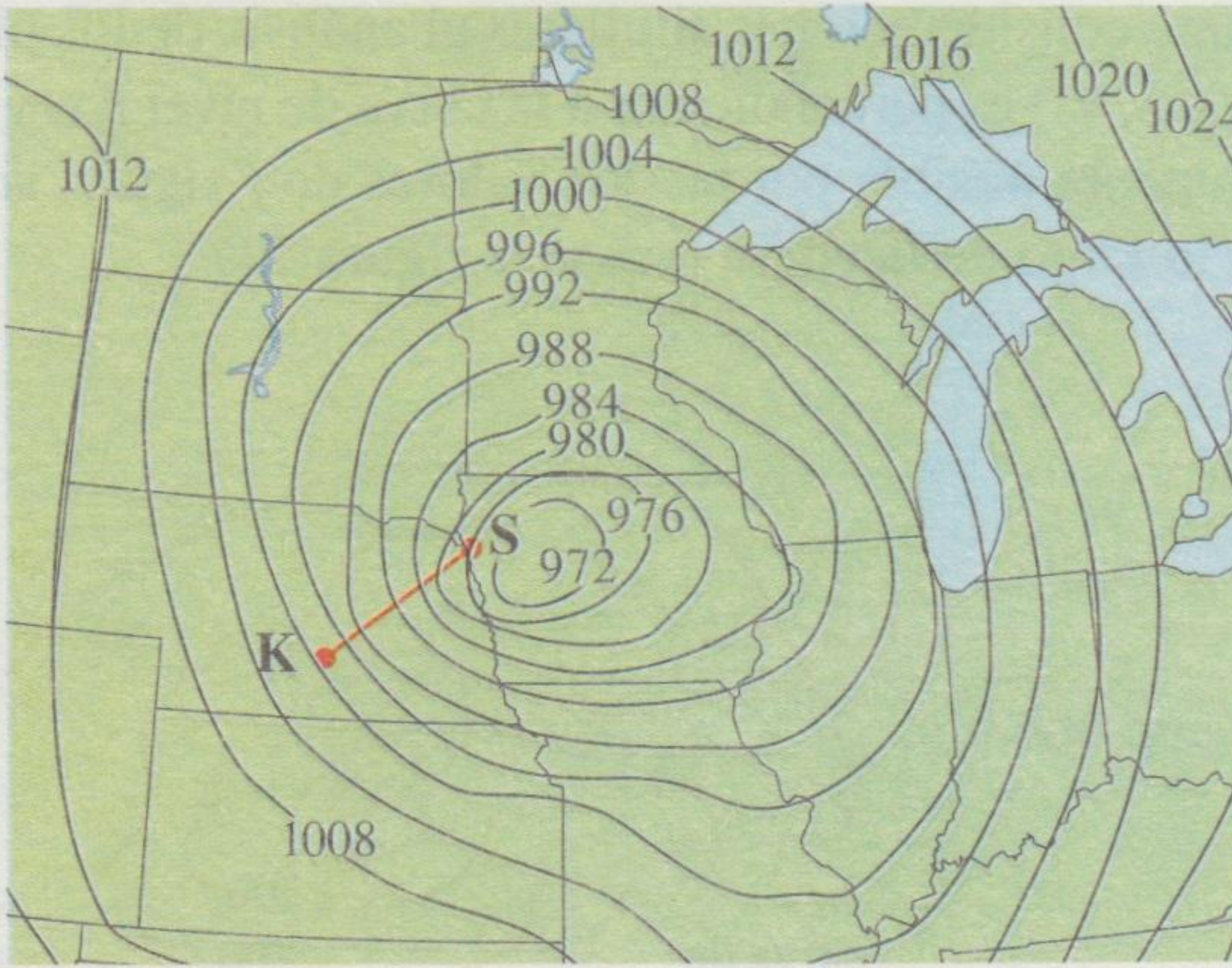


FIGURA 12



4000
4500
5000
LONESOME MOUNTAIN.
A
5500
B
5000
4500
Lonesome Creek

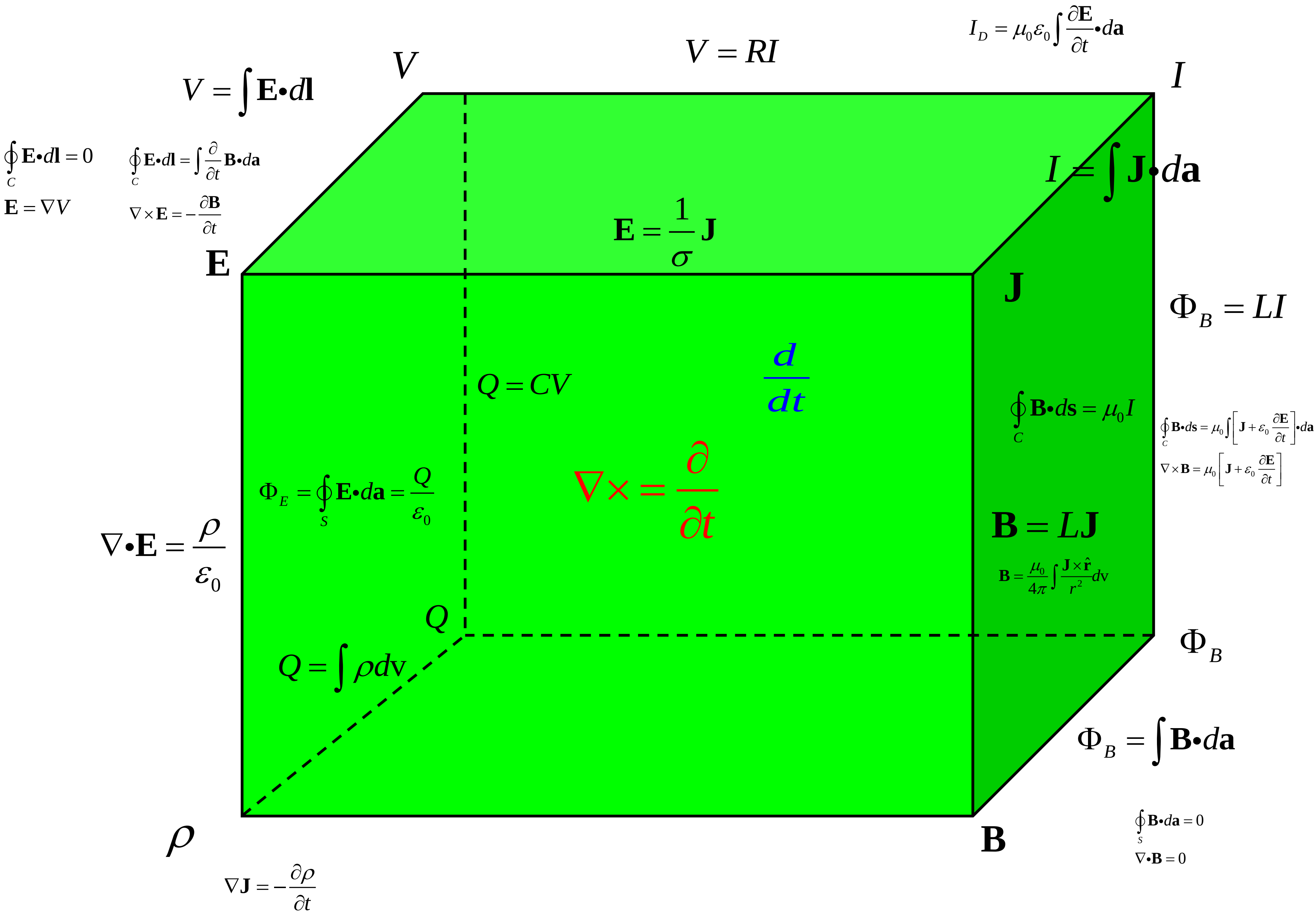


De *Meteorology Today*, 8E por C. Donald Ahrens (2007 Thomson Brooks/Cole).

Aplicacion a escalares y vectores

$$\nabla escalar$$

$$\nabla vector$$



$I_D = \mu_0 \varepsilon_0 \int \frac{\partial \mathbf{E}}{\partial t} \bullet d\mathbf{a}$
$V = RI$
V
I
$V = \int \mathbf{E} \bullet d\mathbf{l}$
$\oint_C \mathbf{E} \bullet d\mathbf{l} = 0$
$\oint_C \mathbf{E} \bullet d\mathbf{l} = \int \frac{\partial}{\partial t} \mathbf{B} \bullet d\mathbf{a}$
$I = \int \mathbf{J} \bullet d\mathbf{a}$
$\mathbf{E} = \nabla V$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\mathbf{E} = \frac{1}{\sigma} \mathbf{J}$
$\mathbf{E}$
$\mathbf{J}$
$\Phi_B = LI$
$\frac{d}{dt}$
$Q = CV$
$\oint_C \mathbf{B} \bullet d\mathbf{s} = \mu_0 I$
$\oint_C \mathbf{B} \bullet d\mathbf{s} = \mu_0 \int \left[\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right] \bullet d\mathbf{a}$
$\nabla \times \mathbf{B} = \mu_0 \left[\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right]$
$\Phi_E = \oint_S \mathbf{E} \bullet d\mathbf{a} = \frac{Q}{\varepsilon_0}$
$\nabla \times = \frac{\partial}{\partial t}$
$\nabla \bullet \mathbf{E} = \frac{\rho}{\varepsilon_0}$
$\mathbf{B} = L\mathbf{J}$
$\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J} \times \hat{\mathbf{r}}}{r^2} dv$
Q
Φ_B
$Q = \int \rho dv$
$\Phi_B = \int \mathbf{B} \bullet d\mathbf{a}$
ρ
$\mathbf{B}$
$\oint_S \mathbf{B} \bullet d\mathbf{a} = 0$
$\nabla \bullet \mathbf{B} = 0$
$\nabla \mathbf{J} = -\frac{\partial \rho}{\partial t}$

$3D$ Vector Gradiente

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

$$\nabla \bullet \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \triangleq \nabla^2 \leftarrow \textit{Laplaciano}$$

$$\nabla^2 \psi = 0 \leftarrow \text{ec. de Laplace}$$

$$\nabla^2 \psi(\mathbf{r}) = 0$$

$$\psi\left(\mathbf{r}, t\right)$$

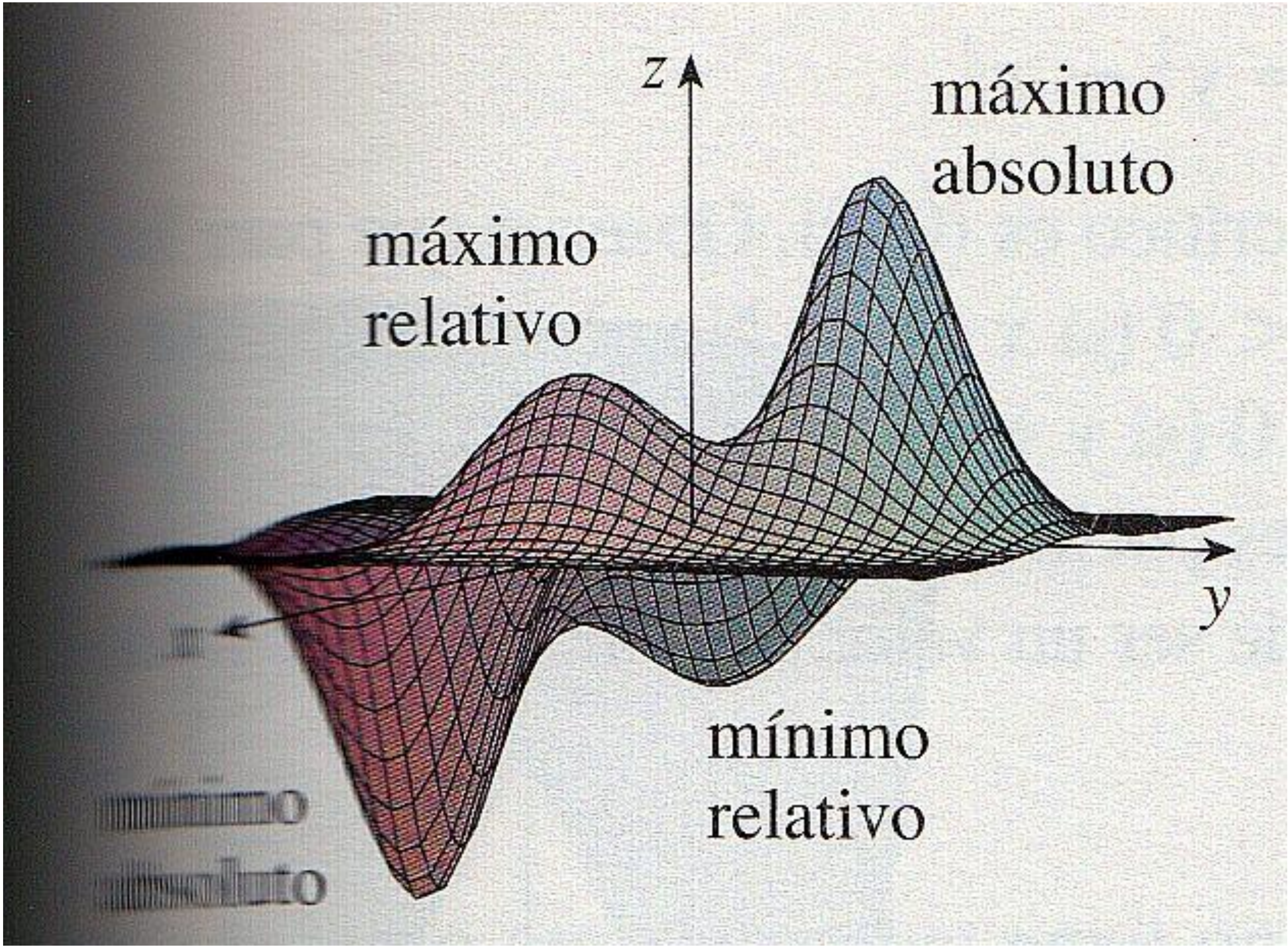
$$\nabla^2 \psi - \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = 0 \leftarrow \textit{ec.} \text{ de onda}$$

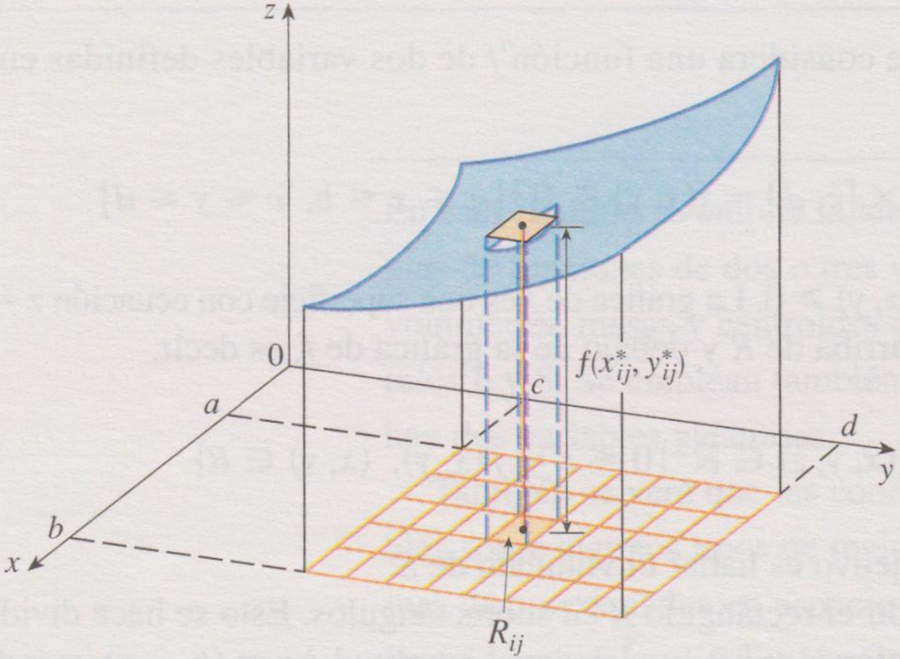
$$\psi\left(\mathbf{r}, t\right) = \cos(\mathbf{r} - \mathbf{v}t)$$

El valo maximo de la derivada direccional $D_{\mathbf{u}} f(\mathbf{r})$ es $\left\| \nabla f(\mathbf{r}) \right\|$

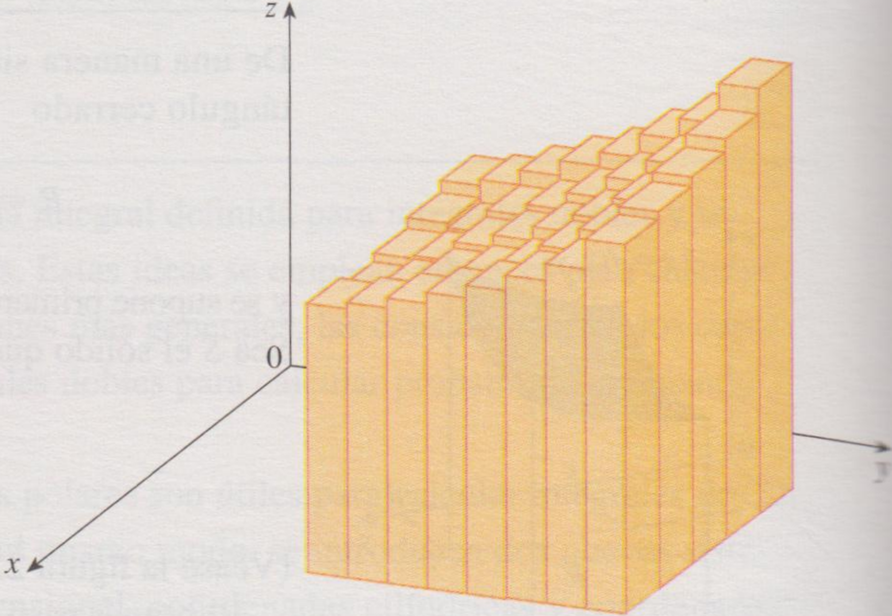
lo cual ocurre cuando $\mathbf{u}$ tiene la misma direccion que el vector gradiente

$$\nabla f(\mathbf{r}) \bullet \mathbf{u} = \left\| \nabla f(\mathbf{r}) \right\| \bullet \left\| \mathbf{u} \right\| \cos\theta = \left\| \nabla f(\mathbf{r}) \right\| \bullet \left\| \mathbf{u} \right\| = \left\| \nabla f(\mathbf{r}) \right\|$$

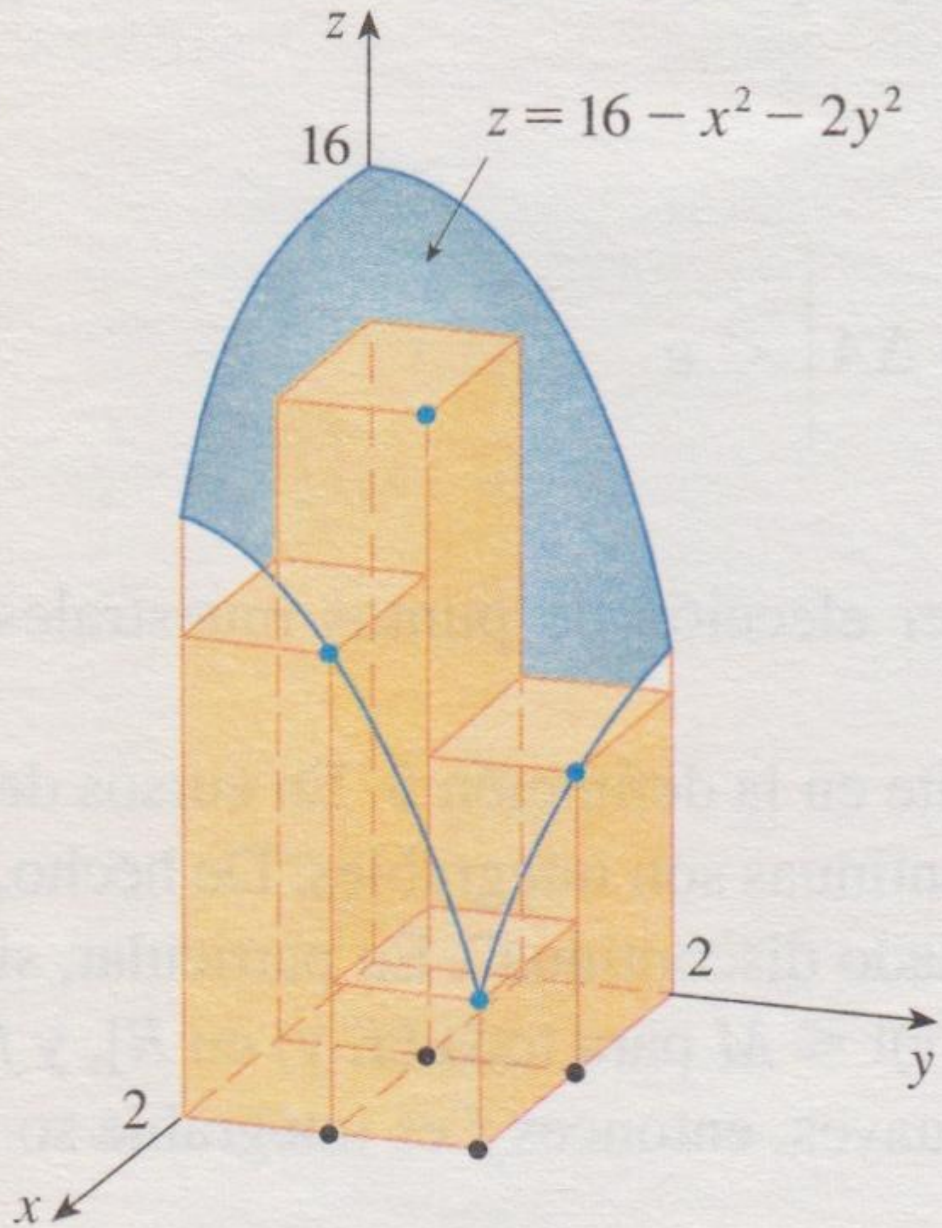

z
máximo
absoluto
máximo
relativo
y
mínimo
relativo

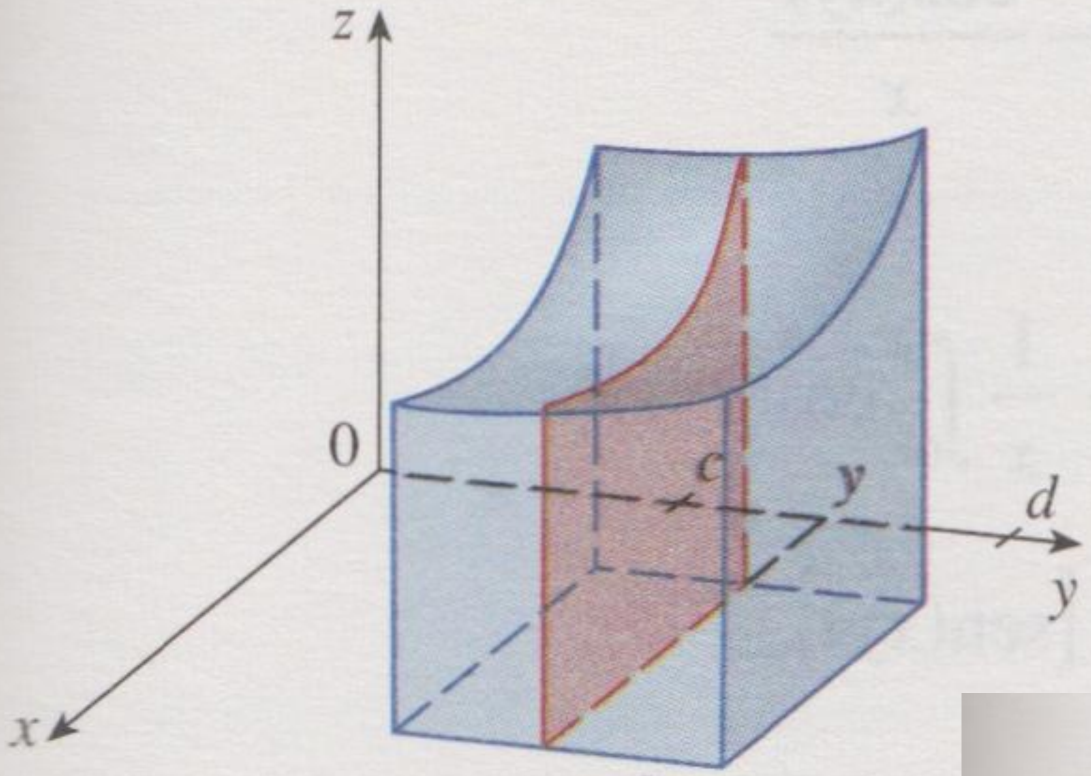


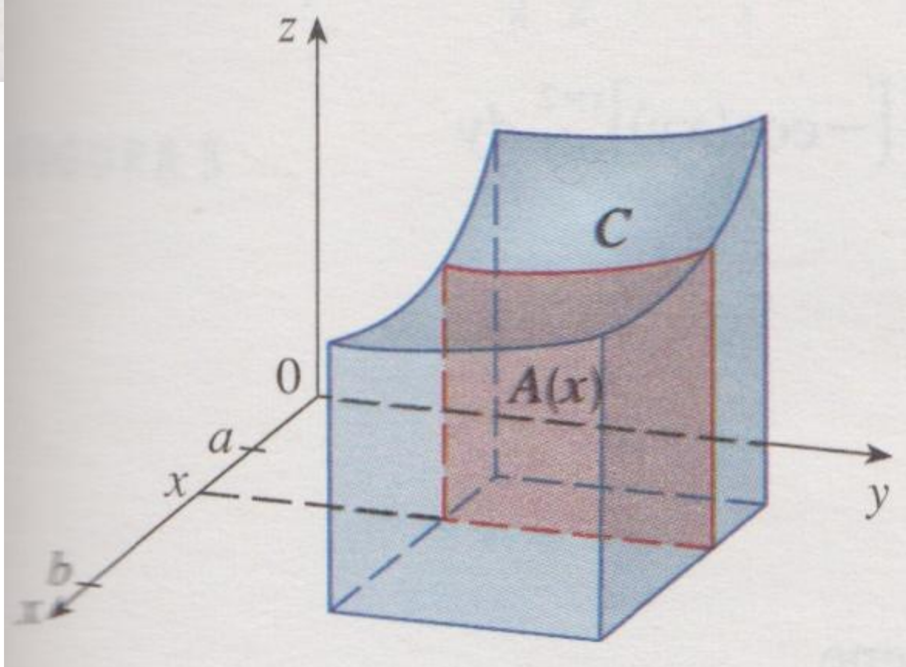
z
0
a
c
d
y
b
x
$f(x_{ij}^*, y_{ij}^*)$
R_{ij}



z
0
x
y


z
$z = 16 - x^2 - 2y^2$
16
2
y
2
x


z
0
c
y
d
y
x


z
C
0
A(x)
a
x
y
b
x

Intergrales a un escalar a un vector